Optimal Design of an "Energy Tower" Power Plant

Eyal Omer, Rami Guetta, Ilya Ioslovich, Per-Olof Gutman, *Senior Member, IEEE*, and Michael Borshchevsky

***Abstract*—The Energy Tower (ET) is a power plant that uses seawater and solar energy accumulated in hot dry desert air to produce electricity. An optimization algorithm was designed and programmed for the design of an ET that also includes water reservoirs, pipes, and pumps. The algorithm is a variant of the block coordinate descent method, where one subproblem is formulated as a linear programming problem, and a second subproblem usually has an analytic solution. If the analytic solution is nonfeasible, a variant of the quasi-Newton algorithm is used instead. The algorithm was tested with input information from the arid area near Eilat, the Red Sea, Israel.**

***Index Terms*—Energy management, optimization methods, power system modeling, power system planning.**

NOMENCLATURE

avg_maint — O and M costs [¢/kW·h].
brine_coef$_i$ — Derivative of the piece-wise linear function $q3(q2)$ in section i.
$C_{\mathrm{elec\_cons}}$ — Cost of electricity [$\$/\mathrm{kW\cdot h}$].
$C_{\mathrm{elec\_prod}}$ — Price for produced electricity [$\$/\mathrm{kW\cdot h}$].
C_{hw} — Hazen–Williams friction coefficient.
$C_{\mathrm{pump}}j$ — Installation cost of pumps j [$/kW].
C_X — Cost of component X per installed unit.
days_in_month(m) — Number of days in month [m].
D_j — Diameter of pipe j [m].
evap_disch — An approximated evaporation rate from the lower reservoir [m^3/s].
Gr$_{\mathrm{1D}}$ — The 1-D model gross power from the ET [MW].
gross_power — Piece-wise linear approximated gross power from the ET [MW].
h — Hour.
h_j — Pumping head of pump j[m].
$h_{\mathrm{loss},j}$ — Head losses in pipe j[m].
h_{res1} — Height difference between the lower reservoir and the water source [m].
L_j — Length of pipe j[m].
m — Month.
N — Number of sections in a piece-wise linear function.
N_{pipes2} — Number of pipes for the spraying water.
p_j — Piece-wise linear approximated pumping power of pump i[MW].
P — Gross turbine power [MW].
pp_i — Pumping power of pump i[MW].
prX — Capitalization coefficient for component X [1/year].
q_{2i} — Spraying discharge at section i[m^3/s].
q_3 — Brine discharge from the ET [m^3/s].
q_{br} — The 1-D model brine discharge from the ET [m^3/s].
Q_j — Discharge capacity of pipe j [m^3/s].
q_{spr} — Spraying discharge [m^3/s].
$V_{\max,j}$ — Maximal allowed velocity of the water inside pipe j[m/s].
Z^* — Maximal objective function of the optimal design problem.
Z_1 — Objective function of subproblem 1.
β_i — Derivative of the piece-wise linear gross power function in section i [MJ/m^3]
$\gamma_{j,k}$ — Derivative of the piece-wise linear pumping power function of pump j in section k [MJ/m^3].
$\eta_{p,j}$ — Pumping efficiency of pump j.
ρ_w — Water density [kg/m^3].

I. INTRODUCTION

THE ENERGY Tower (ET) is a power plant that uses hot dry air and seawater to produce electricity, without air pollution and without emission of greenhouse gases, in arid and hot climates. Unlike other solar energy technologies that require solar collectors and work between 6 and 8 h per day, the ET cools hot atmospheric air and generates a downdraft that works continuously 24 h per day. The power plant is based on a very large cylindrical chimney (see Fig. 1). Water sprayed at the top of the tower partially evaporates, thus cooling the surrounding air from dry-bulb temperature, to close to its wet-bulb temperature. The cool air is heavier, and falls down through the tower. This artificial wind can reach a speed of 80 km/h at the bottom of a 1200 m high tower [1], [9]. Wind turbine generators at the bottom of the tower produce electricity. After the turbines, the air is led through diffusers that decrease its speed, thus decreasing kinetic energy losses and increasing pressure recovery. Different aspects of the ET, including the modeling of the formation of droplets and wind flow are found in [2]–[5] and [9]–[11].

A scheme of the ET system is shown in Fig. 2. Water is pumped from the sea using low-head pumps (P_1) through a pipe (L_1) to the lower reservoir (V_1). Water from the lower reservoir is pumped using the high-head pumps (P_2) to the spraying system at the top of the tower. The air that exits the bottom of the tower is cold, humid, and contains brine droplets. These droplets fall to the ground and are collected by drainage system, conveyed to the brine reservoir (V_2), and from there returned to the sea.

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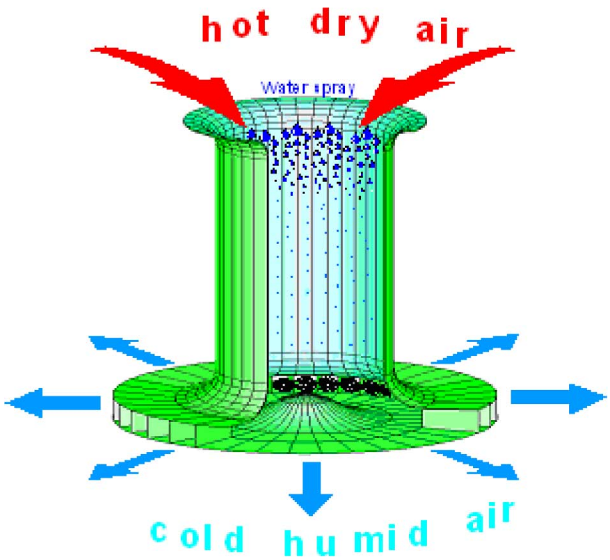

Fig. 1. ET principle of operation.

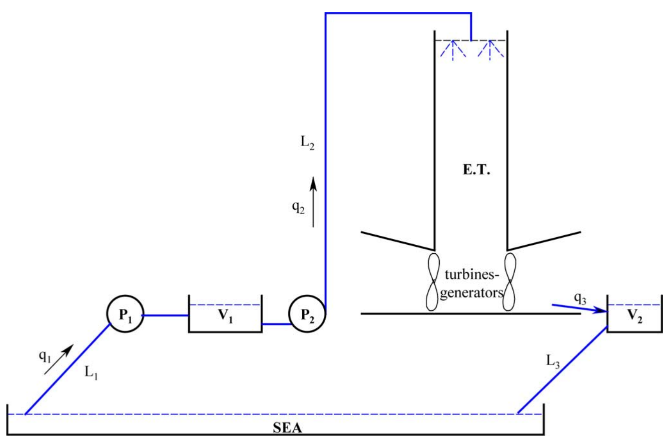

Fig. 2. Scheme of the ET plant. L_i: pipe i; P_i: pumping station i; V_i: reservoir i; q_i: water discharge in pipe i.

The paper is organized as follows. Section III describes the physical model of the ET plant and the optimization algorithm. Section IV presents the optimization results. The disscussion and conclusions are found in Section V, and the Appendix gives the mathematical formulation of the LP problem that is part of the optimization problem. All the calculations have been done for the potential project location near the city of Eilat, Red Sea, Israel.

II. Optimal Design and the Mathematical Model

An optimal design is considered for the expected lifetime of the ET of 30 years, taking into account the cost of installation. It is assumed that the climate is well represented by specially constructed 12 representative days, one day for each month, 24 h a day, the so-called short reference year (SRY) (see [6]). Our model of representative days was suggested by Segal *et al.* [7]. This model gives the values of the temperature and the humidity ratio at the altitude of 900 m in the city of Eilat, for each month of the year, which is represented as 24 h. Hence, the

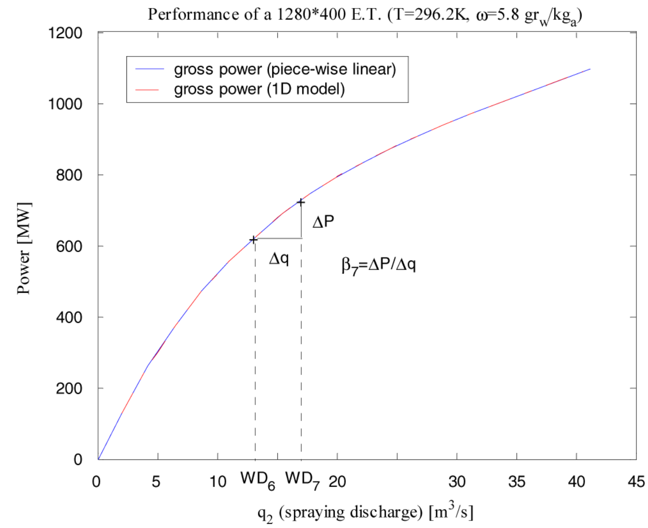

Fig. 3. Gross power from the ET versus the spraying discharge for some specific climate conditions and tower dimensions. The height of the ET is 1280 (meter) and its diameter is 400 (meter). An example of the parameters of the piece-wise linear function is also shown, where β_7 is the derivative of the piece-wise linear function at the seventh linear section, which is defined between the discharges WD_6 and WD_7.

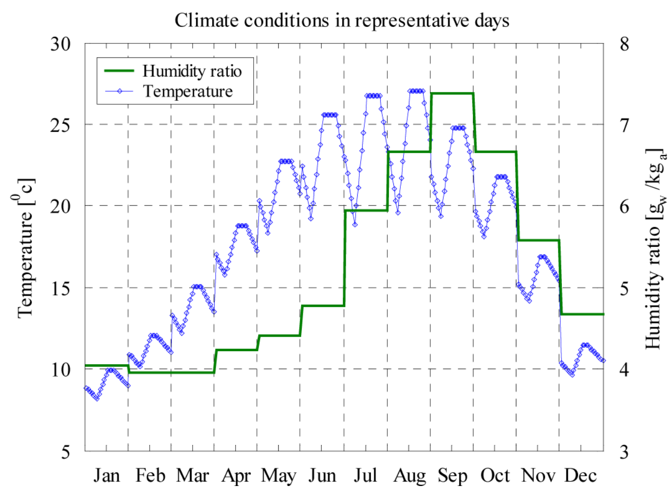

Fig. 4. Climate conditions for the representative 24 × 12 h.

total number of representative, distinct hours in a year is $24 \times 12 = 288$. The height of the ET was chosen to be 1280 m, so the values of the temperatures were reduced according to the model of standard atmosphere, which is a temperature reduction of 0.65 °C for each increasing of 100 m in elevation. The humidity ratio was assumed not to change. The representative day (temperature and humidity ratio for 24 h) of each month is shown in Fig. 4.

Another important assumption is that the electric tariffs and bank discount are constant. The costs of all components are calculated with respect to their specific lifetimes. Hence, it suffices to consider the dynamic model of one year. The objective function is the annual net profit.

The model includes linear and nonlinear constraints and functions of the objective function. Linear equations describe the hourly values of the pumped discharges in the pipes, and the volumes in the reservoirs. The constraints related to the design variables, e.g., installed turbine and pumping power, diameters of pipes and volumes of reservoirs are also taken into account. Mass balances give the dynamic model for $24 \times 12 = 288$ periods.

The gross power of the ET is a function of the discharge of spraying water, the climate conditions, and the turbine coefficient. It is found from the 1-D model [2], [4] as an analytical function. In order to use gross power as a function of spraying discharge in a linear programming (LP) formulation, the function is approximated as the piece-wise linear function

$$\begin{aligned} \text{gross\_power}(m,h) &= \sum_{i=1}^{N_2} \beta_i(m,h) q_{2i}(m,h) \\ \beta_i(m,h) &= \left(\frac{Gr_{1D}(WD_i) - Gr_{1D}(WD_{i-1})}{WD_i - WD_{i-1}} \right)_{m,h} \\ & 0 \le \mathrm{q}_{2i}(m,h) \le WD_i(m,h), \qquad i = 1 \\ & 0 \le \mathrm{q}_{2i}(m,h) \le WD_i(m,h) \\ & \quad - WD_{i-1}(m,h), \quad i = 2, \ldots, \mathrm{N}_2 - 1 \\ & 0 \le \mathrm{q}_{2i}(m,h), \qquad i = \mathrm{N}_2 \\ & \beta_i \ge \beta_{i+1} \end{aligned} \tag{1}$$

where m and h are indices of month and hour, respectively, and gross_power is the piece-wise linearly approximated gross power from the ET, $\mathrm{Gr}_{1\mathrm{D}}$ is the gross power from the ET that is obtained from the 1-D model, N_2 is the number of sections in the piece-wise linear function, and q_{2i} is the spraying discharge at section i (m^3/s), which is defined by the range $[WD_{i-1}, WD_i]$. An example of this function is shown in Fig. 3, where the parameters of the piece-wise linear approximant are also shown for linear section no. 7.

Some part of the sprayed water is not evaporating, and reaches the bottom of the tower as brine water. The brine water is collected in a brine reservoir and returned back to the sea. In order to establish a water dynamic mass balance in the brine reservoir, the brine discharge from the collecting area to the brine reservoir should be estimated. The algorithm to find the brine discharge as a function of the spraying discharge is similar to the algorithm to find the gross power as a function of the spraying discharge. The relation between the brine discharge and the spraying discharge has an opposite trend to the trend of the gross power–spraying discharge relation; the maximal evaporation rate occurs for the minimal amount of spraying water, so the brine discharge rate is minimal for the minimal spraying discharge, and increases as the spraying discharge increases. Thus, the true relation between the brine discharge and the spraying discharge is a nonlinear increasing function with respect to the spraying discharge. In order to include this function in an LP formulation, the function was approximated as a piece-wise linear function with the same sections of q_2 (the spraying discharge) as before. The correspondent piece-wise linear function is

$$\begin{aligned} q_3(m,h) &= \sum_{i=1}^{N_2} \text{brine\_coe}f_i(m,h) q_{2,\mathrm{i}}(m,h) \\ \text{brine\_coef}_i(m,h) &= \left(\frac{qbr_{1D}(WD_i) - qbr_{1D}(WD_{i-1})}{WD_i - WD_{i-1}} \right)_{m,h} \\ & 0 \le q_{2i}(m,h) \le WD_i(m,h), \quad i = 1 \\ & 0 \le q_{2i}(m,h) \le WD_i(m,h) \\ & \quad - WD_{i-1}(m,h), \quad i = 2, \ldots, N_2 - 1 \\ & 0 \le q_{2i}(m,h), \quad i = N_2 \\ & \text{brine\_coef}_i \le \text{brine\_coef}_{i+1} \end{aligned} \tag{2}$$

where $q_3(m,h)$ is the piece-wise linear approximated brine discharge at period (m,h) (cubic meter per second) and $q\ \mathrm{br}_{1\mathrm{D}}$ is the brine discharge that is obtained from the 1-D model.

The *optimization problem* is defined as simultaneously finding the optimal diameters of pipes, water discharges, and pumping power. The relation between these variables is given by

$$pp_j = 10^{-6} \frac{\rho_w g}{\eta_{p,j}} (h_j + h_{\mathrm{loss},j}) \, q_j \tag{3}$$

where pp_j is the pumping power of pump j (megawatt), ρ_w is the water density (kilogram per meter cube), g is the gravity acceleration (meter per second square), $\eta_{p,j}$ is the pumping efficiency of pump j, h_j is the pumping head of pump j (meter), $h_{\mathrm{loss},j}$ is the head losses in pipe j (meter), and q_j is the discharge through pump j (cubic meter per second) . The head losses in a pipe due to friction are estimated according to the widely used Hazen–Williams equation

$$h_{\mathrm{loss}} = \mathrm{L} \times 1.131 \times 10^9 \left(\frac{\mathrm{q}}{\mathrm{C_{hw}}} \right)^{1.852} D^{-4.87} \tag{4}$$

where L is the length of the pipe (meter), q is the water discharge (cubic meter per hour), C_{hw} is the Hazen–Williams friction coefficient, and D is the diameter of the pipe [millimeter], where, for a smooth pipe, $C_{\mathrm{hw}} \in [120], [150]$.

For a given diameter, there is a discharge, called discharge capacity that leads to the maximal allowed water velocity in the pipe. It is found from

$$Q_j = \frac{\pi D_j^2}{4} V_{\max,j} \tag{5}$$

where Q_j is the discharge capacity of pipe j (cubic meter per second), D_j is the diameter of pipe j (meter) and $V_{\max,i}$ (meter per second) is the maximal allowed velocity of the water inside the pipe as recommended by the pipe manufacturer.

Combining (4) and (5) gives

$$\begin{aligned} pp_j(q_j, Q_j) &= 10^{-6} \frac{\rho_w g h_j}{\eta_{p,j}} q_j + \frac{\rho_w g}{\eta_{p,j}} L_j \times 1.131 \\ & \times 10^3 \left(\frac{3600}{C_{hw,j}} \right)^{1.852} \times \left(\frac{\sqrt{4} \times 1000}{\sqrt{\pi V_{\max,j}}} \right)^{-4.87} \\ & q_j^{2.852} Q_j^{-2.435}. \end{aligned} \tag{6}$$

In order to use a compact form of (6), two constants that are related to pumping station i are defined as follows:

$$c_{1,j} = 10^{-6} \times \frac{\rho_w g h_j}{\eta_{p,j}} \tag{7}$$

$$c_{2,j} = \frac{\rho_w g}{\eta_{p,j}} L_j \times 1.131 \times 10^3 \left(\frac{3600}{C_{hw,j}}\right)^{1.852} \left(\frac{2000}{\sqrt{\pi V_{\max,j}}}\right)^{-4.87}. \tag{8}$$

In a problem that involves a given diameter, the pumping power of pump j is only a function of the discharge through the pump and the pipe as

$$pp_j = c_{1,j} q_j + c_{D,j} q_j^{2.852} \tag{9}$$

where $c_{D,j}$ is a constant that depends on the discharge capacity of pipe j, and is calculated as follows

$$c_{\mathrm{D},j} = c_{2,\mathrm{j}} Q_j^{-2.435}. \tag{10}$$

In addition to the simplification of the function, the given diameter also limits the range of discharges to a finite range [0, Q_j], where Q_j is the discharge capacity in pipe j, which is defined by the diameter. These simplifications make it possible to approximate the convex function $p_j\ (q_j)$ as a piece-wise linear function, and use it in an LP formulation. The piece-wise linear function is constructed as

$$\begin{aligned} p_j(m,h) &= \sum_{k=1}^{\mathrm{N_j}} \gamma_{i,k} q_{j,k}(m,h) \\ \gamma_{j,k} &= \frac{\Delta pp_{j,k}}{\mathrm{WD\_SEC}_j} \\ \mathrm{WD\_SEC}_j &= Q_j / N_j \\ 0 \le q_{j,k}&(m,h) \le \mathrm{WD\_SEC}_j \end{aligned} \tag{11}$$

where $p_j\ (m,h)$ is the piece-wise linear approximated pumping power of pump j at period (m,h) (megawatt), $\Delta pp_{j,k}$ is the difference of pumping power of pump j between the end and the beginning of section k (megawatt), N_j is the number of sections for the piece-wise linear function of pump j, and $q_{j,k}$ is the discharge through pump j at section k (cubic meter per second). The range of discharges is divided into N_j equal sections, where the width of each section is $\mathrm{WD\_SEC}_j$ (cubic meter per second).

We notice that the annual profit (annual income minus annual costs) is maximized, and the nonlinear functions included in the annual costs (brine discharge and pumping power) are convex while the nonlinear function included in the annual income (gross power) is concave. Hence, the use of piece-wise linear approximations is valid.

However, the ET system design problem includes a nonlinear function of two variables, e.g., (6). Hence, the optimization problem cannot be formulated directly as an LP problem.

A subproblem, which includes *known* diameters, can, however, be formulated as an LP problem, using the piece-wise linear approximations of the pumping power (11), as well as the piece-wise linear approximations of the gross power and the brine discharge vs. the spraying discharge (1) and (2), respectively. The original optimization problem is, therefore, divided into two subproblems.

1) Finding the maximal annual profit for given discharge capacities of pipe 1 (Q_1) and pipe 2 (Q_2).
2) Finding the optimal Q_1 and Q_2 when all other variables are fixed.

The optimization algorithm used is block coordinate descent [8]. At each step, it finds a local maximum with respect to a group (block) of variables. In our case, one step includes the solution of subproblem 1, and the second step includes the solution of subproblem 2. These steps are repeated until convergence is reached. Subproblem 1 is an LP problem, and was solved by the LP solver of MOSEK (www.mosek.com). The solution of this subproblem contains the optimal discharges for given diameters. For given discharges, the optimal diameters, i.e., the solution of subproblem 2, can then be found analytically. The discharge capacity, or equivalently, the diameter of the pipe has an affect on three components in the objective function: the cost of the pipe, the cost of the pump, and the pumping energy.

The subsystem that includes the pipe (L_1) and pumping station (P_1) obviously influences the rest of the system. However, for given hourly discharges (q_1), changing the diameter has no other effect on the rest of the system than the installed pumping power and the pumping energy, as mentioned earlier. Therefore, for a given vector of discharges (q_1), this subsystem can be separated from the rest of the system and be optimized as follows

The installed pumping power (megawatt) of pumping station 1 is

$$\begin{aligned} P_1(Q_1,q_1) &= 10^{-6} \frac{\rho_w g}{\eta_{p,1}} (h_{\mathrm{res1}} + h_{\mathrm{loss},1}(Q_1, q_{1,\max})) q_{1,\max} \\ &= c_{1,1} q_{1,\max} + c_{2,1} q_{1,\max}^{2.852} Q_1^{-2.435} \end{aligned} \tag{12}$$

where h_{res1} (meter) is the height difference between the lower reservoir and the water source, $h_{\mathrm{loss},1}\ (Q_1, q_{1,\max})$ (meter) is the maximal head losse in pipe L_1 for any given discharge capacity Q_1, which corresponds to the maximal discharge $q_{1,\max}$. The cost of pump 1 (million dollars per year) is:

$$P_1 \cos t(Q_1, q_1) = C\mathrm{pump1} \cdot P_1 \cdot pr\mathrm{PUMP1} \times 10^{-3} \tag{13}$$

where Cpump1 is the installation cost of pump 1 (dollars per kilowatt), P_1 is the installed power of pumping station 1 (megawatt), and *pr*PUMP1 is the capitalization coefficient for pump 1 (1/year). The installation cost of a pipe (dolars per meter) is approximated as the linear function

$$\mathrm{pipe}_i \cos t(Q_i) = C\mathrm{pipe}i_a Q_i + C\mathrm{pipe}i_b. \tag{14}$$

Thus, the cost of pipe 1 (million dollars per year) is

$$\mathrm{pipe}_1 \cos t(Q_1) = (C\mathrm{pipe1}_a Q_1 + C\mathrm{pipe1}_b) L_1 pr L_1 \times 10^{-6} \tag{15}$$

where L_1 is the length of pipe 1 (meter) and *pr*L_1 is the capitalization coefficient for pipe 1 (1/year). The hourly pumping power (megawatt) of pump 1 is

$$p_1(m,h)=10^{-6}\frac{\rho_w g}{\eta_{p,1}}h_{\text{res}1}q_1(m,h)+c_{1,1}q_1^{2.852}(m,h)Q_1^{-2.435} \quad (16)$$

The annual cost of pumping energy for pump 1 (million dollars per year) is

$$p1\cos t=\sum_{m=1}^{12}\sum_{h=1}^{24}[C\text{elec\_cons}(m,h)\cdot \cdot p_1(m,h)\times 10^{-3}\ \text{days\_in\_month}(m)] \quad (17)$$

where Celec_cons(m,h) (dollars per kilowatt hour) is the cost of electricity during the period (m,h), days_in_month (m) is the number of days in month $m, p_1\ (m,h)$ (megawatt) is the hourly pumping power and 10^{-3} is a conversion coefficient between units.

The optimal discharge capacity $Q_1{*}(q_1)$ (cubic meter per second) is found from zeroing the derivative of the cost function with respect to the discharge capacity

$$\frac{\partial(\text{pipe1}\cos t+P1\cos t+p1\cos t)}{\partial Q_1}=0 \quad (18)$$

where

$$\frac{\partial \text{pipe1}\cos t}{\partial Q_1}=C\text{pipe1}_a L_1 prL_1\times 10^{-6}$$

$$\frac{\partial P1\cos t}{\partial Q_1}=-2.435\cdot C\text{pump}_1\cdot pr\text{PUMP1}\times 10^{-9}\cdot c_{1,1}\cdot q_{1,\max}^{2.852}\cdot Q_1^{-3.435}$$

$$\frac{\partial p1\cos t}{\partial Q_1}=-2.435\times 10^{-9}c_{1,1}Q_1^{-3.435}$$

$$\sum_{m=1}^{12}\text{days\_in\_month}(m)\sum_{h=1}^{24}C\text{elec}(m,h)q_1^{2.852}(m,h)$$

The solution to (18) is

$$\text{Q}_1^*(q_1)=\left(\frac{k_{L1}}{k_{p1}(q_1)}\right)^{\frac{-1}{3.435}} \quad (19)$$

where

$$k_{L1}=C\text{pipe}_{1\_a}\cdot L_1 prL1\times 10^{-6}$$

$$k_{p1}(q_1)=2.435\times 10^{-9}\cdot c_{1,1}\cdot\left[C\text{pump}_1\cdot pr\text{PUMP1}\cdot q_{1,\max}^{2.852}+\sum_{m=1}^{12}\text{days\_in\_month}(m)\times\sum_{h=1}^{24}C\text{elec}(m,h)\cdot q_1^{2.852}(m,h)\right]$$

The second derivative of the cost function (pipe1cost $+P1$cost $+p1$cost) is obviously always positive; therefore, Q_1^* is the minimizer of this cost function. A similar procedure is followed in order to find the optimal discharge capacity of pipe 2 for a given spraying discharge $Q_2^*\ (q_2)$ (cubic meter per second), leading to

$$\text{Q}_2^*(q_2)=\left(\frac{k_{L2}}{k_{p2}(q_2)}\right)^{\frac{-1}{3.435}} \quad (20)$$

where

$$k_{L2}=C\text{pipe}_{2\_a}L_2\cdot prL2\times 10^{-6}$$

$$k_{p2}(q_2)=2.435\times 10^{-9}c_{1,2}\left[C\text{pump}_2\cdot pr\text{PUMP2}\cdot q_{2,\max}^{2.852}+\sum_{m=1}^{12}\text{days\_in\_month}(m)\times\sum_{h=1}^{24}C\text{elec}(m,h)q_2^{2.852}(m,h)\right]$$

$$\text{c}_{1,2}=N\text{pipes}_2\cdot\frac{\rho_1\cdot \text{g}}{\eta_{p2}}\cdot L_2\cdot 1.131\times 10^9\left(\frac{3600}{N\text{pipes}_2\cdot C\text{hw}_2}\right)^{1.852}\cdot\left(\frac{2000}{\sqrt{\pi\cdot V\max_2\cdot N\text{pipes}_2}}\right)^{-4.87}$$

and $N\text{pipes}_2$ is the number of pipes for the spraying water.

The values $Q_1(q_1)$ and $Q_2(q_2)$ represent the minimizer of the *unconstrained* cost function, and either of them might be smaller than its respective value in the previous iteration that leads to an infeasible solution. The infeasibility is due to an increase of head losses when the diameter is decreased, which leads to an increase in pumping power. As this increase holds for all hours of the solution, the maximal pumping power will be increased above the installed pumping power. The maximal water velocity might also increase above the maximal allowed water velocity. Although the final solution of the algorithm is feasible, an infeasible solution of some iteration is not accepted, since then optimality cannot be proved. Thus, if either one of the discharge capacities $Q_1(q_1)$ and/or $Q_2(q_2)$ is smaller than its value from the previous iteration, the block coordinate descent algorithm is abandoned, and the optimal solution is found by finding the optimal pair of discharge capacities Q_1 and Q_2, which maximize the objective function of subproblem 1

$$Z^*=\max_{Q_1,Q_2} Z_1(Q_1,Q_2) \quad (21)$$

where Z^* is the maximal objective function of the optimal design problem and Z_1 is the objective function of subproblem 1. Each solution (including the optimal) of subproblem 1 is feasible, as it is a solution of the LP problem. The optimal (Q_1,Q_2) is found by a quasi-Newton algorithm, for which we used the MATLAB function "fminunc." The algorithm to find the global maximum is shown in Fig. 5.

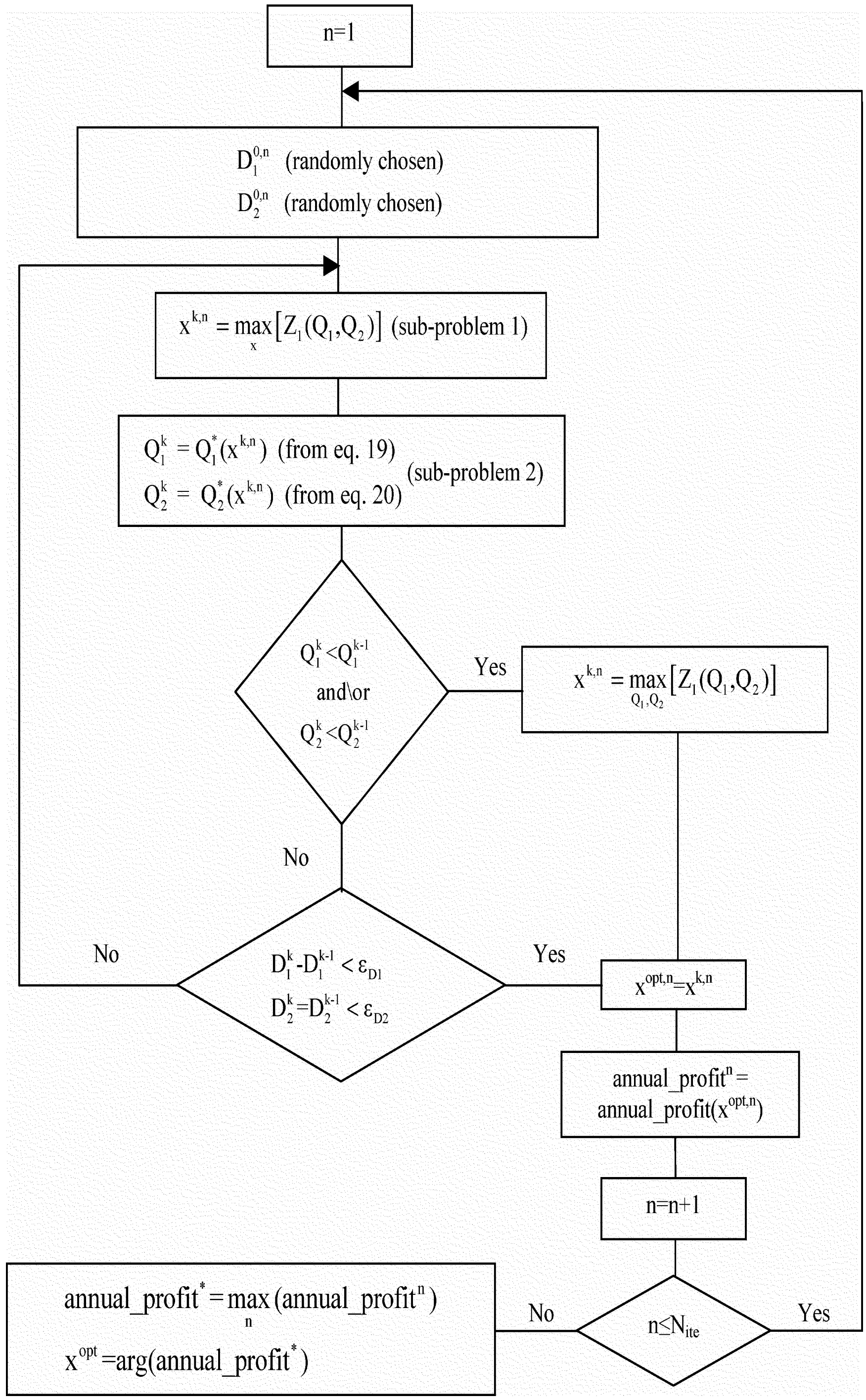

Fig. 5. Optimization algorithm for the design of an ET plant.

III. Optimization Results

The model for the optimal design of an ET system was run with input parameters that are relevant to a considered potential project. The main input parameters are listed in Table I. The values of the optimal design variables are listed in Table II. The installation cost of the components sums to 49% of the total installation cost, while the remaining 51% is the installation cost of the structure, which is constant and not a design variable. The optimal discharge capacities of the pipes are listed with their corresponding diameter. The maximal discharge in pipe 1 is 18.2 (cubic meter per second), which is 90% of the discharge capacity of pipe 1, and the maximal spraying discharge (q_2) is 14.2 (cubic meter per second), which is about 50% of the discharge capacity of pipe 2. As expected, the installation cost of the reservoirs is much smaller than the cost of the other components. A summary of performance and economy parameters of the ET is listed in Table III. The average ratio between the pumping power and gross power is about 44% and the minimal ratio is about 35%, which is achieved on a June afternoon,

TABLE I
MAIN INPUT PARAMETERS FOR THE MODEL IMPLEMENTATION

Parameter	Units	Value
Height of the tower	m	1280
Diameter of the tower	m	400
Distance of the site from the sea	km	40
Installation cost of the ET structure	M$	450
Cost of pumps	$/kW	400
Cost of turbines-generators	$/kW	320
Annual interest rate	%	5

TABLE II
OPTIMAL DESIGN OF AN ET PLANT

Design variable	Symbol	Units	Value
Lower reservoir - volume	V_1	10^3 m^3	261
Brine reservoir - volume	V_2	10^3 m^3	35
Pipe 1 - diameter	D_1	mm	2927
Pipe 2 – diameter (each of 8 pipes)	D_2	mm	1214
Pipe 3 - diameter	D_3	mm	1188
Pumping station 1 - installed pumping power	P_1	MW	22
Pumping station 2 - installed pumping power	P_2	MW	226
Installed turbine power	PTUR	MW	629

when the best climate conditions prevail. An interesting question is the following: What is the maximal net power that can be potentially produced by an ET at a specific site, and what is the difference in the performance, compared to the performance that is obtained by designing the system for maximal net profit. It is found that a reduction of 25% in the *installed turbine power* is obtained in the optimal design compared to the installed turbine power of 836 (megawatt), which corresponds to the potential maximal net power. The *maximal potential net power* is obtained on a June afternoon, and it is 455 (megawatt) in comparison to 411 (megawatt) net power obtained in the optimal design. As the slope of the gross power with respect to spraying discharge is much higher than the slope of the net power for spraying discharge values lower than q^*_{spr}, which is the spraying discharge maximizer of net power, the reduction in the maximal net power in the optimal design is only 9% compared to the maximal potential net power. The approximate 8 (cubic meter per second) reduction in spraying discharge, which corresponds to this reduction in gross and net power, is the main reason to the reduction of 35% in the total installed pumping power, as it reduces to about 132 (megawatt) of the installed power of the high-head pumps. Although the installed turbines power is reduced, the reduction of pumping power and the efficient use of the turbines lead to an annual net energy production, which is only 3% less than its value when the design is based on the policy of maximal net power. It is obvious from this comparison that a design, which is based on the criterion of potential maximal net power is very wasteful.

TABLE III
ANNUAL PERFORMANCE AND ECONOMY SUMMARY OF AN ET PLANT
(PARAMETERS THAT ARE MARKED WITH *NET CORRESPONDS TO A POLICY OF MAXIMAL NET POWER)

Parameter	Units	Value
Annual gross energy production	10^9 kWh/year	3.713
Annual pumping energy consumption	10^9 kWh/year	1.621
Annual net energy production	10^9 kWh/year	2.092
Maximal net power (June afternoon)	MW	411
Annual water consumption	MCM/year	347
Annual brine water returned to sea	MCM/year	91
$\frac{\text{maximal gross power}}{\text{gross power(*net)}}$	%	75
$\frac{\text{maximal pumping power}}{\text{pumping power(*net)}}$	%	65
$\frac{\text{maximal net power}}{\text{net power(*net)}}$	%	91
$\frac{\text{annual net energy production}}{\text{annual net energy production(*net)}}$	%	97
Initial investment	M$	885.2
Annual investment return (% of total annual costs)	M$/year	66.6 (33%)
Annual energy cost (% of total annual costs)	M$/year	116.0 (57%)
Annual O&M (% of total annual costs)	M$/year	20.6 (10%)
Total annual costs	M$/year	203.3
Average production cost	¢/kWh	4.17
Total annual incomes	M$/year	272
Annual net profit	**M$/year**	**69**
Internal Rate of Return (30 years)	%	15.1

The annual costs of the plant include the initial investment, the operation and maintenance (*O* and *M*) costs, and the pumping energy (electricity) costs. The average production cost of the

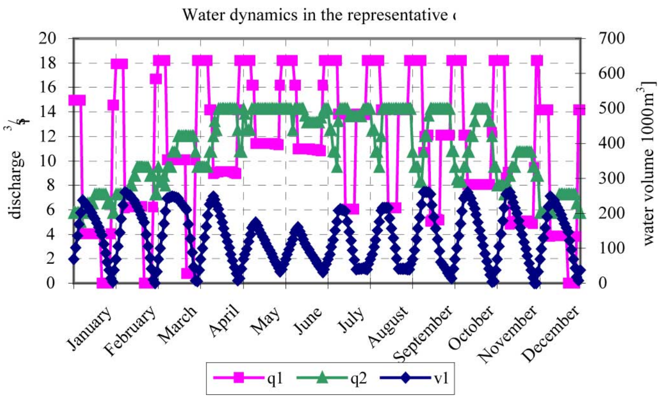

Fig. 6. Discharges of q_1 (from the sea to the lower reservoir), water volume in the lower reservoir v_1, and q_2 (spraying discharge) during the representative days. Each month is represented by 24 h.

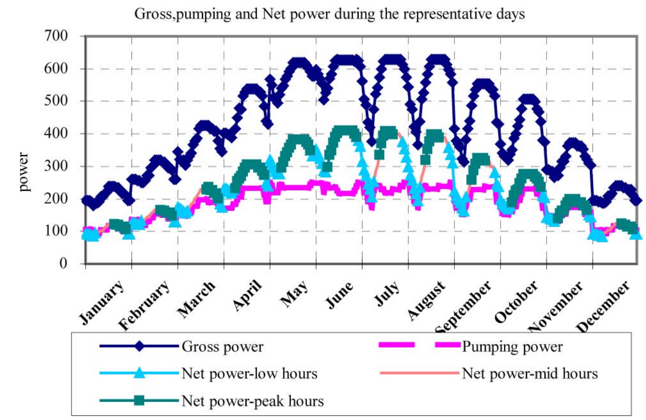

Fig. 7. Gross, pumping, and net power during the representative days. Each month is represented by 24 h.

plant in this example is 4.17 (dollars per kilowatt hour), which is similar to the production costs in fossil- fuel-based power stations [9]. The annual income from the plant is 272 (million dollars) and the total annual costs are about 203 (million dollars), which lead to an annual profit of 69 (million dollars). The corresponding internal rate of return (IRR) is 15.1%.

The operational discharges q_1 (discharge from the sea to the lower reservoir) and q_2 (spraying discharge), and the corresponding water volume in the lower reservoir (v_1) are shown in Fig. 6. Each column in the plot includes the representative 24 h of a month, starting from 01:00 AM at the left line of the column, and ending with 24:00 PM at the right line of the column. The electricity tariff table (not shown) defines that low demand hours start at 23:00 and end at 6:00 or 7:00 AM, depending on the month.

The use of low-cost electricity is clearly seen in Fig. 6, where pumping discharge from the sea is done with full capacity during low demand hours most of the year, except in the winter, when much less water is needed for spraying. The pumping from the sea during low demand hours brings the water volume in the lower reservoir to its maximal value, and the water from the reservoir is used during the day for spraying. The small amount of spraying water in the winter, compared to the rest of the year, makes it possible to pump from the sea only during low demand hours. During the rest of the year, seawater is also pumped during mid demand hours. Our result shows that this procedure is more economical than an increase in the discharge capacity of pipe 1, which would have enabled a larger amount of water to be pumped from the sea during low demand hours. As best climate conditions for producing power prevail in the summer (June–August), the produced gross power equals the installed power of the turbines, as can be seen in Fig. 7, where the gross power, pumping power, and net power during the representative days are shown. The constant gross power during the afternoon in the months May–October is obtained because the climate model assumes constant climate conditions during these periods of time (see Fig. 4), and, moreover, the electricity tariff (not shown) does not change during these hours. It can be seen in Figs. 6 and 7 that the gross power in June afternoon equals the installed turbines power, even though smaller spraying discharge is used, compared to July and August, when the same maximal gross power is obtained by a spraying discharge that equals the spraying discharge capacity. This reduction of spraying discharge in June is due to the best climate conditions that exist on a June afternoon, which makes it possible to produce the same gross power as in August by a smaller spraying discharge. This is the reason why the ratio of pumping power to gross power is minimal during this period of time. It would be feasible to spray a higher discharge during June afternoon, but as it would lead to a gross power that is larger than the installed power of the turbines, it would only require additional pumping power, without leading to additional income.

IV. Discussion and Conclusion

In this paper, a model and an optimization algorithm for the optimal design of an ET system was formulated and programmed. The objective function of the optimization algorithm is to maximize annual net profit from the system. The optimization algorithm is based on "block coordinate descent," where one step includes a fixation of the diameters of two pipes (D_1 and D_2), and the other step includes a fixation of all other variables. The subproblem with the fixed diameters was formulated as an LP problem, where most functions were approximated as linear functions. The functions of the pumping power, the gross power from the ET, and the brine discharge from the ET, which are highly nonlinear with respect to the discharge, were approximated as piece-wise linear functions. The other subproblem, which includes finding the optimal diameters when all other variables are fixed, is solved analytically by zeroing the gradient of the objective function. The algorithm stops when the change in the diameters is smaller than some accepted error. In case an infeasible solution is obtained, a quasi-Newton algorithm is used in order to find the maximum of the problem over the subdomain of the two diameters (D_1 and D_2).

APPENDIX I

MATHEMATICAL FORMULATION OF THE LP PROBLEM

A. General Form

The general form of the optimization problem is

$$\begin{aligned}&\min_x Z = c^T x\\&\text{subject to:}\\&Ax \le \text{b}\\&A_{\text{eq}}x = b_{\text{eq}}\\&lb \le x \le ub\end{aligned} \tag{22}$$

where Z is the objective function to be minimized, c is a vector of costs (c_i is the cost of variable i per unit of the variable), x is a vector of the variables, lb is a vector of the lowest values that can be assigned to the variables, and ub is a vector of the highest values that can be assigned to the variables.

The matrix A and the vector b include the parameters of the matrix form of the inequality constraints

$$\sum_{j=1}^{N\text{vars}} a_{ij}x_j = b_{\text{eq},j}, \quad i = 1, \ldots N_{\text{eq}}$$

The matrix A_{eq} and the vector b_{eq} include the parameters of the matrix form of the equality constraints

$$\sum_{j=1}^{N\text{vars}} a_{\text{eq},ij}x_j = b_{\text{eq},j}, \quad i = 1, \ldots, N_{\text{eq}}$$

where N_{vars} is the number of variables, N_{ineq} is the number of inequality constraints and N_{eq} is the number of equality constraints.

B. Formulation of LP1

1) Variables and Limits ($lb_j \le x_j \le ub_j$):

a) Design variables:

$0 \le V_i \le V_{i,\max}$ – volume of reservoir i ($i = 1, 2$)[1000 m^3]

$0 \le P_1 \le \infty$ – installed power of pump i ($i = 1, 2$)[MW]

$0 \le Q_3 \le \infty$ – installed discharge capacity of pipe 3 [m^3/s]

$0 \le \text{PTUR} \le \infty$ – installed turbines power [MW]

2) (Approximated) Operation Variables [at period (m, h)]:

$0 \le v_i(m,h) \le \infty$ – water volume in reservoir i [1000 m^3]

$0 \le p_i(m,h) \le \infty$– pumping power of pump $i (i = 1, 2)$ [MW]

$0 \le q_{1,x}(m,h) \le \frac{Q_1}{10}$– the part of the discharge in pipe1 that corresponds to the part x ($x = 1, \ldots, 10$) in the piece-wise linear function p_1 (q_1) [m^3/s]

$0 \le q_3(m,h) \le \infty$– the discharge in pipe 3 [m^3/s]

$0 \le q_{2,1}(m,h) \le WD_1$ – the part of the spraying discharge that corresponds to the first part in the piece-wise linear function gross_power (q_2) [m^3/s]

$0 \le q_{2,i}(m,h) \le WD_i - WD_{i-1}$ – the part of the spraying discharge that corresponds to the ith part in the piece-wise linear function gross_power (q_2), $i = 2, \ldots, 14[m^3/s]$

$0 \le q_{2,15}(m,h) \le Q_2 - WD_{14}$ – the part of the spraying discharge that corresponds to the 15th part of the piece-wise linear function gross_power (q_2) [m^3/s]

3) Objective Function: The objective function is to maximize annual profit. The problem was formulated as a minimization problem, as follows:

$$\min Z^1 = \text{annual costs} - \text{annual incomes} = \sum_i Z_i^l \tag{23}$$

where Z^1 is the annual profit (with a minus sign) (1000 dollars per year), and Z_i^1 are reservoirs installation

$$Z_1^1 = \sum_{i=1,2} C\text{res}_i pr\text{RES}_i V_i$$

where $C\text{res}_i$ is the installation cost of reservoir i (dollars per meter cube) and *pr*RES_i is the capitalization coefficient for reservoir i (1/year). Pumps i installation ($i = 1, 2$)

$$Z_2^1 = C\text{pump}_i \cdot pr\text{PUMP}_i \cdot P_i \tag{24}$$

Pipe 3 installation

$$Z_3^1 = C\text{pipe3}_a \cdot L_3 \cdot prL_3 \cdot Q_3 \tag{25}$$

Turbines installation

$$Z_4^1 = C\text{tur} \cdot pr\text{TUR} \cdot P\text{TUR} \tag{26}$$

where *C*tur is the installation cost of the turbines–generators (dollars per kilowatt) and *pr*TUR is the capitalization coefficient for the turbines–generators (1/year).

Pumping energy costs for pumps i ($i = 1, 2$)

$$Z_5^1 = \sum_m \text{days\_in\_month}(m) \sum_h C\text{elec\_cons}(m,h) \cdot p_i(m,h) \tag{27}$$

Generated power income and *O* and *M* costs

$$\begin{aligned}Z_6^1 = \sum_m \text{days\_in\_month}(m) \sum_h \sum_{k=1}^{15} \beta_k\, [&\text{avg\_maint}\\ &- C\text{elec\_prod}(m,h)]\, q_{2,k}(m,h)\end{aligned} \tag{28}$$

where avg_maint is the *O* and *M* costs [¢/kWh] and *C*elec_prod (m,h) is the price for produced electricity at period (m,h) [\$/kWh].

4) Constraints: The hourly water volume in the reservoirs cannot exceed the capacity of the reservoir

$$v_i(m,h) - Vi \le 0, \quad i = 1, 2. \tag{29}$$

Hourly pumping power of pumps i ($i = 1, 2$) cannot exceed the installed pumping power

$$p_i(m,h) - P_i \le 0. \tag{30}$$

The water velocity in each pipe cannot exceed the maximal water velocity, which is recommended by the manufacturer of the pipes. This implies that the hourly water discharge cannot exceed the discharge capacity. For pipe 1 and 2, the hourly discharge is divided in sections, so the discharge is the sum of the discharges in all sections

$$\sum_{k=1}^{N_i} q_{i,k}(m,h) \le Q_i, \quad i=1,2 \tag{31}$$

$$q_3(m,h) - Q_3 \le 0. \tag{32}$$

The hourly generated gross power from the ET is evaluated from the piece-wise linear function (1), and it cannot exceed the installed power of the turbines. Hence

$$\sum_{k=1}^{15} \beta_k(m,h) q_{2k}(m,h) - \text{PTUR} \le 0. \tag{33}$$

The piece-wise linear approximation of the gross power

$$\text{gross\_power}(m,h) = \sum_{k=1}^{15} \beta_k(m,h) q_{2k}(m,h). \tag{34}$$

Mass balance in the reservoirs gives

$$v(t+\Delta t) - v(t) = [q_{\text{in}}(t) - q_{\text{out}}(t)]\,\Delta t. \tag{35}$$

In this model, the discharges are constant during 1 h, so $\Delta t =$ 3600 s. The units of water volume in the reservoirs are [1000 m^3] and the units of discharges are (cubic meter per second). In order to conserve the consistency of units, the right-hand side of (35) is divided by 1000. Hence, the factor that multiplies the discharges in the mass balance equations is 3.6 (3600/1000).

The mass balance in the lower reservoir is, therefore

$$v_1(m,h+1) - v_1(m,h) + 3.6\left[\sum_{j=1}^{15} q_{2j}(m,h) - \sum_{k=1}^{10} q_{1,k}(m,h)\right] = -3.6\ \text{evap\_disch} \tag{36}$$

where evap_disch is an approximated evaporation rate from the lower reservoir (cubic meter per second). The mass balance in the brine reservoir is

$$v_2(m,h+1) - v_2(m,h) + 3.6[q_3(m,h) - \sum_{k=1}^{15} \text{brine\_coef}_k(m,h) q_{2,k}(m,h)] = 0. \tag{37}$$

It should be noticed that the model assumes a representative 24 h for each month. This means that the two equations earlier ensures a mass balance during the day, and between the final hour of the month and the first hour of the next month, but they do not ensure a mass balance between midnight and 1:00 during a month. In order to ensure this mass balance, the following two sets of equations were added

$$v_1(m,1) - v_1(m,24) + 3.6\left[\sum_{j=1}^{15} q_{2j}(m,24) - \sum_{k=1}^{10} q_{1,k}(m,24)\right] = -3.6\ \text{evap\_disch}, \quad m = 1,\ldots,12 \tag{38}$$

$$v_2(m,1) - v_2(m,24) + 3.6 \cdot \left[q_3(m,24) - \sum_{k=1}^{15} \text{brine\_coef}_k(m,24) q_{2,k}(m,24)\right] = 0, \quad m = 1,\ldots,12 \tag{39}$$

The piece-wise linear function of the pumping power of pumps i ($i = 1, 2$) gives

$$p_i(m,h) - \sum_{k=1}^{N_i} \gamma_{i,k} q_{i,k}(m,h) = 0. \tag{40}$$

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