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"Energy Tower" combined with pumped storage and desalination: Optimal design and analysis

E. Omer[a], R. Guetta[b], I. Ioslovich[b,*], P.O. Gutman[b], M. Borshchevsky[b]

[a]Tahal Ltd, Tel-Aviv, Israel

[b]Faculty of Civil and Environmental Engineering, Technion-Israel Institute of Technology, 32000 Haifa, Israel

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Abstract

The "Energy Tower" (ET) is a power plant project which uses hot dry air and seawater to produce electricity. An optimized design of a system that is a combination of an ET, pumped storage and seawater desalination plant is considered. A model set covering each subsystem, and results of the optimized design for a project in the area of Eilat are presented. The additional benefit from combining the systems comes from an efficient use of the energy in the brine water coming from the desalination process, and from using pumped storage in an unconventional way. The benefits of the combined system lead to an increase of 14% in the annual net profit, compared to the sum of profits from optimally designed stand-alone systems.

Keywords: Energy Tower; Power system optimization; Optimal design; Desalination; Pumped storage

1. Introduction

The Energy Tower (ET) includes a new technology which uses hot dry air and seawater to produce electricity. Different aspects of the ET, including the modeling of the formation of droplets and wind flow are found in [1–4].

Pumped storage is a method to store energy by converting electrical energy to potential energy by pumping water to an upper reservoir. The potential energy can be converted again to electrical energy when desired by letting the water flow down through a hydro-electric turbine generator. A desalination plant is used to desalinate a seawater by the principle of reverse osmosis (RO). These three systems can be installed as stand-alone systems; however, by combining them together, additional profit can be obtained due to savings in initial investments and energy usage, compared to the sum of profits from the stand-alone systems. This paper includes descriptions of models, formulation of an optimization problem for the optimal design and analysis of results obtained for a planned combined system in the Eilat area, Israel, near the Red Sea.

2. Description of the subsystems

The ET technology makes it possible to produce electricity in arid and warm climate zones, utilizing the abundance of hot dry air as a source of production of environmentally clean energy. Unlike conventional solar technologies which require solar collectors which are sensitive to cloudy days and work only 6–8 h per day, the ET does not require direct solar radiation. In the ET, hot dry air is cooled thus generating a downdraft wind flow that works continuously 24 h per day. The plant is based on a very large hollow cylindrical structure, hereinafter: "tower". Seawater sprayed at the top of the tower partially evaporates, thus cooling the surrounding air from dry-bulb temperature to near its wet-bulb temperature. The cool air is heavier, and falls down through the tower. This artificial wind can reach a speed of 80 km/h at the bottom of a tower of 1200 m height, as shown in [5,6]. Wind turbines generators are installed at the bottom of the tower. Powered by the artificial wind they produce the electricity. After passing the turbines, the air is led through diffusers

*Corresponding author. Tel.: +972 4 8294254; fax: +972 4 8295696.
E-mail address: agrilya@tx.technion.ac.il (I. Ioslovich).

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Notations

avg_maint O&M costs [¢/kWh]
B_{loss} the power loss on the booster pump [W]
brine_coef$_i$ derivative of the piece-wise linear function $q3(q2)$ at section i [dimensionless]
C_{brine} the salt concentration in the brine water [g_s/kg_w]
C_{elec_cons} cost of electricity [$/kWh]
C_{elec_prod} price for produced electricity [$/kWh]
C_{feed} the salt concentration in the feed water [g_s/kg_w]
C_{hw} Hazen–Williams coefficient
C_p the salt concentration in the product (desalinated) water [g_s/kg_w]
C_{pump_i} installation cost of pumps i [$/kW]
C_{spr} the salt concentration in the spraying water [g_s/kg_w]
C_X cost of component X per installed unit
days_in_month(m) number of days in month m
D_i diameter of pipe i [m]
evap_disch an approximated evaporation rate from the lower reservoir [m^3/s]
Gr_{1D} the 1D model gross power from the ET [MW]
gross_power piece-wise linear approximated gross power from the ET [MW]
h index of the hour
h_i pumping head of pump i [m]
h_{desal} the head at the inlet of the membranes [m]
$h_{loss,j}$ head losses in pipe j [m]
h_{loss_memb} the head loss on the membranes [m]
h_{res1} height difference between the lower reservoir and the water source [m]
h_{res2} height difference between the lower reservoir and the upper reservoir [m]
L_i length of pipe i [m]
m index of the month
N number of sections in a piece-wise linear function [dimensionless]
N_{pipes_2} number of pipes for the spraying water
p_i piece-wise linear approximated pumping power of pump i [MW]
P_{in} the power at the inlet to the turbine (or pump) [W]
pp_i pumping power of pump i [MW]
P_{out} either the required pumping power or the generated power from the turbine [W]
$PREV_{2a}$ installed power of reversible pumps [MW]
prX capitalization coefficient for component X [1/year]
PTUR installed turbines power [MW]
Q_{brine_sec} the discharge of the brine water from the desalination plant [m^3/s]
q_i discharge in pipe i [m^3/s]
q_{br} the 1D model brine discharge from the ET [m^3/s]
q_{brine} the discharge of brine water from the desalination plant [m^3/s]
Q_{des_sec} the discharge of desalinated water [m^3/s]
Q_{feed_sec} the discharge of feed water [m^3/s]
Q_i discharge capacity of pipe i [m^3/s]
q_{spr} spraying discharge [m^3/s]
R recovery rate [dimensionless]
RT_{loss} the power loss on the recovery turbine [W]
SR the salt rejection [dimensionless]
$V_{max,i}$ maximal allowed velocity of the water inside the pipe [m/s]
Z^* maximal objective function of the optimal design problem
Z_1 objective function of subproblem 1

Greek

β_i derivative of the piece-wise linear gross power function at section i [MJ/m^3]
$\gamma_{i,k}$ derivative of the piece-wise linear pumping power function of pump i at section k [MJ/m^3]
$\Delta h_{booster}$ the head difference on the booster pump [m]
Δh_{rec_tur} the head difference on the recovery turbine [m]
η_b the efficiency of the booster pump [dimensionless]
$\eta_{p,i}$ pumping efficiency of pump i [dimensionless]
η_T the efficiency of the recovery turbine [dimensionless]
ρ_{brine} the density of the brine water [kg/m^3]
ρ_w water density [kg/m^3]

Abbreviations

ET "Energy Tower"
IRR internal rate of return
LP linear programming
O&M operation and maintenance
RO reverse osmosis
SRY short reference year

which decrease its speed, thus decreasing losses of kinetic energy and increasing pressure recovery. The general scheme of the ET principle of operation is presented in Fig. 1.

The pumped storage and seawater RO desalination plants are based on known technologies that are used world wide, and are well described in the literature, see [11].

3. The water cycle in the stand-alone systems and in the combined system

A scheme of the stand-alone ET plant is shown in Fig. 2. Water is pumped from the sea using low head pumps (P_1), through a pipe (L_1) to the lower reservoir (V_1). Water from the lower reservoir is pumped using the high head pumps

(P_2) to the spraying system at the top of the tower. The air that comes out from the bottom of the tower is cold, humid and contains brine droplets. The brine droplets fall on the ground and the brine is conveyed to the brine reservoir (V_3), and from there it is returned to the sea by gravity.

A scheme of the stand-alone pumped storage plant is shown in Fig. 3. Water is pumped from the lower reservoir (V_1) to the upper reservoir (V_2) using reversible pumps ($PREV_{2a}$) during low-demand hours. The water from the upper reservoir flows by gravity to the lower reservoir during peak hours, operating the reversible pumps as water turbines generators. Water is pumped from the sea by the low head pumps (P_1) in order to compensate the water loss due to evaporation.

A scheme of the stand-alone RO desalination plant is shown in Fig. 4. Water is pumped from the sea using low head pumps (P_1), through a pipe (L_1) to the reservoir (V_1), where primary treatment is performed. Water from the reservoir is pumped using the high head pumps (P_2) to the membranes array. The brine from the membranes array is returned to the sea through recovery turbines generators (REC_TUR), which recover some of the pumping energy.

A scheme of a system combined of an ET, pumped storage and a seawater desalination plant is shown in Fig. 5. Since water from the lower reservoir (V_1) can be pumped simultaneously to the upper reservoir (V_2), to the desalination plant (RO) and to the ET, the required pumping power of pumping station 2a is higher than the required installed power of reversible pumps, so regular pumps (P_{2a}) are installed in parallel to the reversible pumps ($PREV_{2a}$). Water from the upper reservoir can flow down pipe 3 and split into two different discharges: q_{2_t} and q_4. Thus, water from the upper reservoir (V_2) can be used during peak hours in two ways:

1. generating power by the reversible pumps by letting the water flow to the lower reservoir (V_1) through the reversible pumps ($PREV_{2a}$) like a hydro-electric power station (discharge q_{2_t});
2. generating power in the ET by spraying the water at the top of the ET. The water flows down pipe 3 and through

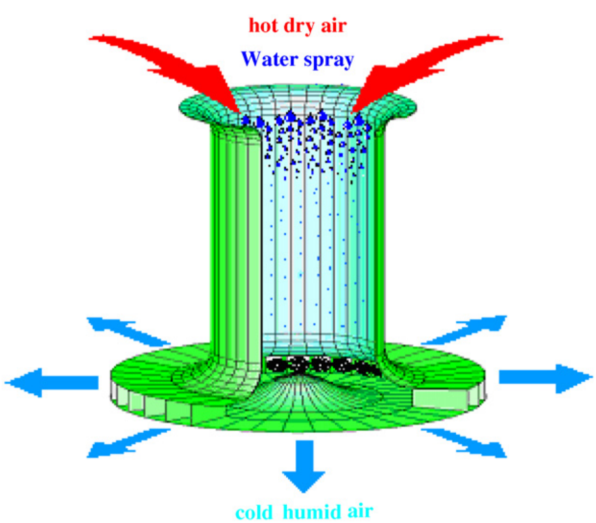

Fig. 1. The ET principle of operation.

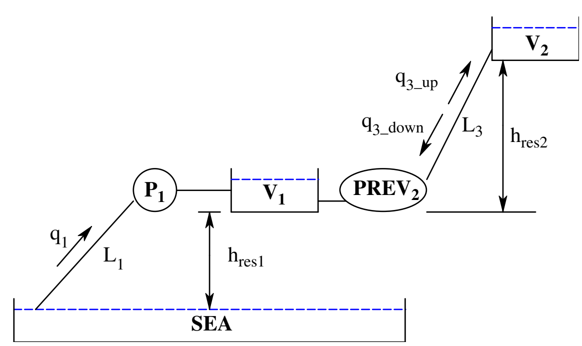

Fig. 3. A scheme of the stand-alone pumped storage plant.

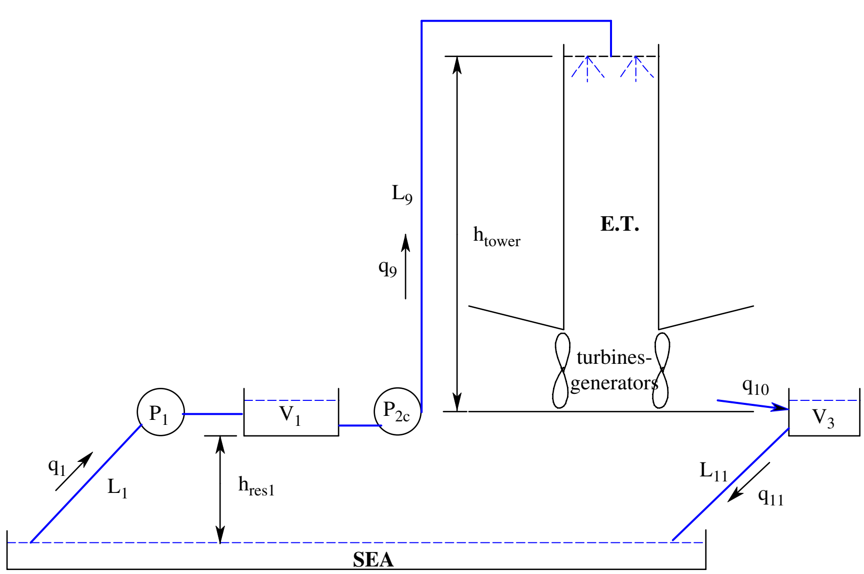

Fig. 2. A scheme of the stand-alone ET.

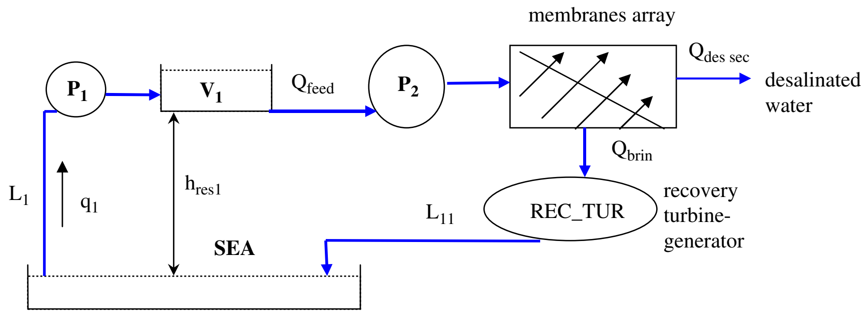

Fig. 4. A scheme of the stand-alone seawater desalination plant.

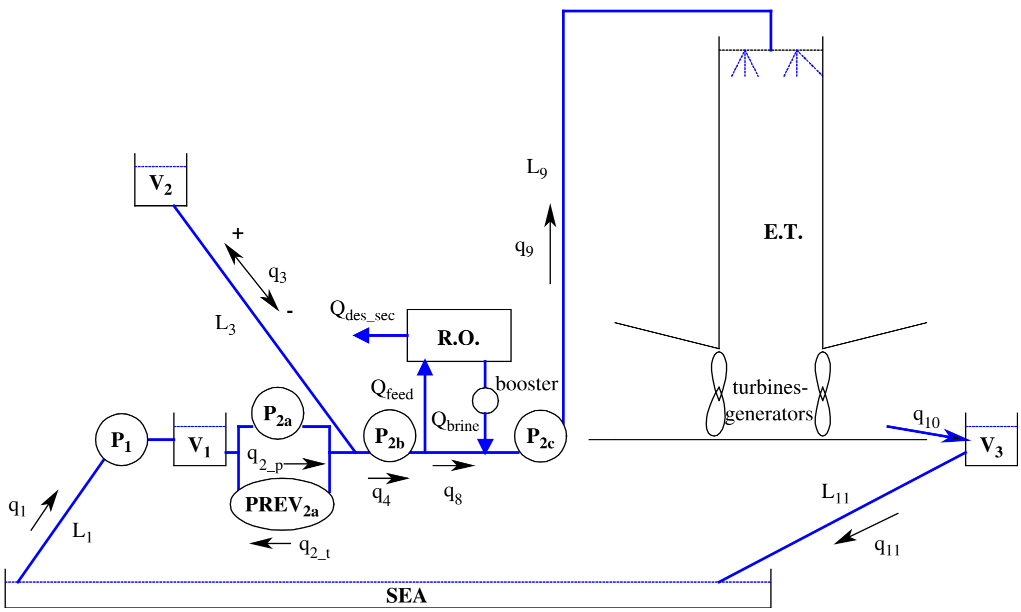

Fig. 5. Scheme of the combined system that includes an ET, pumped storage and a desalination plant.

the pipes 4, 8 and 9. The water of the upper reservoir is delivered to the top of ET together with the water from the lower reservoir during low-demand hours, so the pumping power requirement at peak hours is reduced, and the net power during peak hours is increased.

These two options can coexist, and the optimal decision of what portion of the water will be used in the ET and what portion will be used in the reversible pumps at each representative hour is obtained in the solution of the algorithm. Water in line 4 has the head required for the desalination plant inlet. Some portion of the water is actually directed to the desalination plant, using pump P_{2b} to produce additional head needed for RO, while usually most of the water is directed to pumping station P_{2c}, which lifts the water to the top of the ET. Due to the ET height (1000–1300 m) the necessary head is higher than the desalination head (~700 m). The brine water from the desalination plant in the combined system is not returned to the sea through a recovery turbine. Instead, a booster pump is used in order to increase the head of the RO brine water to the head of the feed water, and then it is mixed with seawater and pumped to the spraying system at the top of the ET.

4. The models for optimal design

The optimization model was formulated for each of the three sub-systems, as well as for the combined system. All these four models have been run with the same input parameters in order to model a situation where the systems are either installed in the same site as stand-alone systems or as a combined system.

An optimal design is considered for the expected lifetime of the system components, taking into account the cost of installation. It is assumed that the climate is well represented by specially constructed 12 representative days, 1 day per month, 24 h a day, a so-called short reference year (SRY), see [7,8]. The approach of SRY was chosen due to its limited data set (288 representative hours) that leads to a moderate size optimization problem. Another important assumption is that the electricity tariffs and bank discount are constant. The costs of all components are calculated with respect to their specific lifetimes. Hence, it suffices to consider the dynamic model of 1 year. The objective function is the annual net profit.

The optimization algorithm that was used for all systems is a variant of block (group) coordinate descent. The block coordinate descent algorithm finds at each step a local maximum with respect to a group of the variables (see e.g. [9]). The block coordinate descent algorithm is very well suited to the optimal design problem of this system, since the problem can be divided into two subproblems (i.e. two groups of variables), where the solution of each of the subproblems can be found very fast. The general idea was to approximate non-linear functions as piece-wise linear functions which makes it possible to formulate the problem as a linear programming (LP) problem, with different problems for each one of the systems—see the Appendix. However, there is one function that makes it impossible to pose the problem as one LP problem for all the variables, since it is a non-linear function of two unknown variables. This function is the pumping power, which is a function of the water discharge and the diameter of the pipe, both of them being decision variables. The problem was divided, therefore, into two subproblems. One subproblem includes fixed diameters of pipe (1) and pipes (9), i.e. pipe 1 in all four systems, and pipes 1 and 9 in the systems that include an ET-see Figs. 2–5. The second subproblem includes only these diameters as variables, while all other system components are fixed, as well as the operational policy, i.e. the water discharges. The first subproblem is formulated as an LP problem, and the optimal diameters in the second subproblem are found by low-dimensional optimization. A full description of the algorithm for the stand-alone ET is given in [10]. The algorithm for all four systems (stand alone and combined) is the same, except for the LP problems that are solved in each iteration of the block coordinate decent, see the LP formulation in the Appendix.

Main assumptions and formulas:

1. All four systems (stand alone and combined) are connected to an electricity grid. The pumping energy is supplied from the grid, and generated energy from the ET and the reversible pumps is supplied to the grid. The systems can consume and supply electricity simultaneously. Although the ET can supply its own pumping energy, the connection to the grid makes it possible to start of the system, and also enables the exclusive use of pumped storage when bad climate condition during the winter might make it unprofitable to use the ET.
2. The head loss in the pipe to the upper reservoir (L_3) is assumed to be constant, and depends only on the length of the pipe, as follows:
$$h_{\text{loss},3} = 1.1\,L_3/1000. \tag{1}$$
Here $h_{\text{loss},3}$ is the head loss in pipe 3 [m] and L_3 is the length of pipe 3 [m].
3. The desalination plant capacity is an input parameter to the model, and the desalination discharge is assumed to be constant during a month. The recovery rate and the salt rejection are assumed to be constant. These parameters are defined as follows, see e.g. [11]:
$$R = \frac{Q_{\text{des\_sec}}(m,h)}{Q_{\text{feed\_sec}}(m,h)}, \tag{2}$$
$$\text{SR} = 1 - \frac{C_{\text{p}}}{C_{\text{feed}}}. \tag{3}$$
Here R is the recovery rate [dimensionless], $Q_{\text{des\_sec}}$ is the discharge of desalinated water [m^3/s], $Q_{\text{feed\_sec}}$ is the discharge of feed water [m^3/s], SR is the salt rejection [dimensionless-], C_{p} is the concentration of salt in the product (desalinated) water [g_s/kg_{water}] and C_{feed} is the concentration of salt in the feed water [g_s/kg_{water}]. The operational variables and parameters are followed by "(m,h)", indicating the hour h in the representative day of month m.

The use of the brine water from the desalination plant in the combined system is the main reason for the energy saving compared to the stand alone subsystems. The energy losses in the recovery turbine are much higher than the energy losses in the booster pump. These losses are found from

$$\text{RT}_{\text{loss}} = P_{\text{in}} - P_{\text{out}} = \rho_{\text{brine}} g \Delta h_{\text{rec\_tur}} q_{\text{brine}} (1 - \eta_{\text{T}}), \tag{4}$$

$$B_{\text{loss}} = P_{\text{out}} - P_{\text{in}} = \rho_{\text{brine}} g \Delta h_{\text{booster}} q_{\text{brine}} \left(\frac{1}{\eta_{\text{b}}} - 1\right), \tag{5}$$

where RT_{loss} is the power loss in the recovery turbine [W], B_{loss} is the power loss in the booster pump [W], P_{in} is the power at the inlet to the turbine (or pump) [W], P_{out} is either the required pumping power or the generated power from the turbine [W], ρ_{brine} is the density of the brine water [kg/m^3], $\Delta h_{\text{rec\_tur}}$ is the head difference on the recovery turbine [m], $\Delta h_{\text{booster}}$ is the head difference on the booster pump [m], q_{brine} is the discharge of brine water from the desalination plant [m^3/s], η_{T} is the efficiency of the recovery turbine [dimensionless] and η_{b} is the efficiency of the booster pump [dimensionless]. By dividing Eq. (4) by Eq. (5), the ratio between the energy losses can be found. Assuming $\eta_{\text{T}} = \eta_{\text{b}} = \eta$, the resulting ratio is

$$\frac{RT_{\text{loss}}}{B_{\text{loss}}} = \frac{\Delta h_{\text{rec\_tur}}}{\Delta h_{\text{booster}}} \eta. \tag{6}$$

The head differences are found from

$$\Delta h_{\text{rec\_tur}} = h_{\text{desal}} - h_{\text{loss\_memb}}, \quad \Delta h_{\text{booster}} = h_{\text{loss\_memb}}, \tag{7}$$

where h_{desal} is the head at the inlet of the membranes [m] and $h_{\text{loss\_memb}}$ is the head loss over the membranes [m]. The head difference over the recovery turbine is much higher than the head difference over the booster pump. Assuming that the inlet head to the membranes is 700 [m], the head loss over the membranes is 50 [m] with an efficiency of 85%. The energy losses when using a recovery turbine is

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about 11 times higher than the energy losses when using the booster pump. However, there is a disadvantage of spraying the RO brine water. The increase of salt concentration of the spraying water leads to a decrease of the generated power from the ET. The concentration of the spraying water is an input to the one-dimensional ET model [2,3]. For a stand-alone ET, the concentration of the spraying water is the same as the concentration of the seawater. In the combined system, the concentration of salt in the spraying water is found from

$$C_{spr}(m,h) = \frac{q_8(m,h)C_{sea} + Q_{brine\_sec}(m,h)C_{brine}}{q_9(m,h)}, \quad (8)$$

where C_{spr} is the salt concentration in the spraying water [g_s/kg_w], Q_{brine_sec} is the discharge of brine water from the desalination plant [m^3/s], and C_{brine} is the salt concentration in the brine water [g_s/kg_w]. The discharge q_8 is the discharge that bypasses the desalination plant, see Fig. 5, and it is found from

$$q_8(m,h) = q_9(m,h) + Q_{des\_sec}(m,h) - Q_{feed\_sec}(m,h). \quad (9)$$

The concentration of the brine water is found from [12]:

$$C_{brine} = \frac{C_{feed}}{(1-R)^{SR}}. \quad (10)$$

The discharge of the brine water from the desalination plant is

$$Q_{brine\_sec} = Q_{feed\_sec}(1-R). \quad (11)$$

5. Results

The algorithm for the optimal design of the stand-alone systems and the combined system that includes an ET, a desalination plant and pumped storage was programmed in the MATLAB environment, and implemented with data from a potential site in the Eilat area, Israel. The main input parameters are listed in Table 1. Table 2 shows the optimal design of the combined system, and the results for these systems as stand-alone systems. Table 3 shows the summary of the performance of the stand alone and the combined systems. Table 4 shows the summary of the economy parameters of the stand alone and the combined systems. The number in parentheses in these tables is the sum of the sizes of the same parameter in all systems, in case stand-alone systems were installed.

The lower reservoir V_1 in the combined system is smaller compared to the stand-alone systems, since it does not have to contain all the water from the upper reservoir V_2, as in the stand-alone pumped storage. Only some amount of the water flows through the reversible pump $PREV_{2a}$ to the lower reservoir, while some amount of the water from the upper reservoir is used as spraying water, see Fig. 5. Recall the explanations in Section 3. Since some amount of the water is returned to the lower reservoir, this amount of water does not have to be pumped from the sea. So the discharge capacity of pipe 1 (Q_1) and pumping station P_1, see Fig. 5, in the combined system are smaller compared to the stand-alone system. Another reason for the decrease of pipe 1 and pumping station 1 is the combining of the desalination plant. The brine water from the desalination plant which is returned back to the sea in the stand-alone desalination plant is sprayed at the top of the ET in the combined system. The additional spraying water makes it possible to generate more power with no extra pumping power, but it requires more powerful high head pumps (pumps P_{2b} and P_{2c}) and wider high head pipes (pipes 9) to the upper reservoir V_2. The installed turbine power is also increased in the combined system due to the increase of the generated power to pumping power ratio. The total installed turbine power is about the same for the combined system as for the stand-alone systems; however, due to the

Table 1
Main input parameters for the model implementation

Parameter	Units	Value
Height of the tower	m	1280
Diameter of the tower	m	400
Distance of the site from the sea	km	40
Height of the upper reservoir	m	500
Capacity of the desalination plant	million m^3/year	20
Recovery rate	%	50
Installation cost of the ET structure	m$	450
Cost of pumps	$/kW	400
Cost of turbines-generators	$/kW	320
Annual interest rate	%	5

Table 2
The optimal design of the components of the stand-alone and the combined systems

Symbol	Units	E.T.	p.s.	desl	E.T. + p.s. + desl
V_1	$10^3\,m^3$	261	1000	17	780 (1278)
V_2	$10^3\,m^3$	–	1000	–	1000
V_3	$10^3\,m^3$	35	–	–	67
Q_1	m^3/s	20.2	0.2	3.1	21.5 (23.3)
Q_3	m^3/s	–	39.7	–	40.4
Q_9	m^3/s	27.8	–	–	30.1
Q_{11}	m^3/s	3.3	–	0.6	3.7 (3.9)
P_1	MW	22	0.1	2.7	23.1 (24.8)
P_{2a}	MW	–	–	–	113
$PREV_{2a}$	MW	–	235	–	170
TUR_{2a}	MW	–	168	–	121
P_{2b}	MW	–	–	10.5	40
P_{2c}	MW	226	–	–	121
REC_TUR	MW	–	–	3.6	–
$P_{booster}$	MW	–	–	–	0.4
P_{total}	MW	248	235.1	13.2	467.5 (496.3)
PTUR	MW	629	–	–	669

The number in parentheses is the sum of the sizes of the same component in all systems, in case stand-alone systems were installed. Legend: E.T.—ET, p.s.—pumped storage, desl—desalination plant.

Table 3
A summary of the performance of the stand-alone and the combined systems

Parameter	Units	E.T.	p.s.	desl	E.T. + p.s. + desl
Gross energy production	kWh/year × 10^9	3.713	0.427	–	3.911 (4.140)
Pumping energy consumption	kWh/year × 10^9	1.621	0.602	0.073	1.974 (2.296)
Net energy production	kWh/year × 10^9	2.092	−0.175	−0.073	1.937 (1.844)
Water consumption	million m^3/year	347	1.5	40	379 (388.5)
Mean spraying discharge	m^3/s	10.98	–	–	11.38
Guaranteed net power at peak hours in January	MW	114	167	–	281

The number in parentheses is the sum of the sizes of the same parameter in all systems, in case stand-alone systems were installed. Legend: E.T.—ET, p.s.—pumped storage, desl—desalination plant.

Table 4
A summary of the economics of the stand-alone and the combined systems

Parameter	Units	E.T.	p.s.	desl	E.T. + p.s. + desl
Initial investment	m$	885	140	44	1099 (1069)
Annual investment return	m$/year	66.6	12.7	3.4	84.3 (82.6)
Annual energy costs	m$/year	116.0	14.9	4.8	109.5 (135.7)
Annual O&M costs	m$/year	20.6	2.4	4.2	25.3 (27.2)
Total annual costs	m$/year	203.3	29.9	12.4	219.1 (245.6)
Total annual incomes	m$/year	272.1	45.5	13	315.9 (330.6)
Annual net profit	m$/year	68.8	15.6	0.6	96.8 (85.1)
IRR (30 years)	%	15.1	20.1	8.2	16.3 (15.5)

The number in parentheses is the sum of the sizes of the same parameter in all systems, in case stand-alone systems were installed. Legend: E.T.—ET, p.s.—pumped storage, desl—desalination plant.

increase of generated power to pumping power ratio in the combined system, the portion of the ET turbines in the combined system is increased. Since energy that is produced from the ET is clean, this portion is supposed to increase even more if a bonus for clean energy is considered. In the combined system, pipe 3 is wider than in the stand-alone pumped storage, since this increase of the pipe makes it possible to supply high discharges of water both to the ET and to the reversible pumps during peak hours.

Although the combination of pumped storage increases the annual gross energy production compared to a stand-alone ET, the gross production from the combined systems is less the sum of gross production from the stand-alone systems. The reason for that is the increase of turbine power of the ET and the decrease of reversible pump power in the combined system; reversible pumps enable high generated power, but they consume higher pumping power than they are able produce. By combining pumped storage and a desalination plant with an ET, a significant increase in the generated power from an ET is enabled by a small increase in the pumping power, so by shifting the generated power from the reversible pumps to an ET, the gross production is reduced, but the pumping energy consumption is reduced even more, which leads to a higher net energy production. The other reason for the increase of net energy production is the more efficient use of the brine water energy in the combined system as discussed in the previous section.

Our analysis has been performed from the point of view of the net profits of the whole project. The question that we studied with respect to the desalination plant was: will the combination of other subsystems with the RO desalination plant be profitable? The answer surely depends on the external economical parameters, among them the price of the desalinated water. This price was assumed to be 65 ¢/m^3, which is a realistic price; compare with the projected price of 53 ¢/m^3 for the water in the Ashkelon desalination plant at the beach of the Mediterranean Sea in Israel. Our calculations showed, however, that when the price is at the level of 41 ¢/m^3 which is a break-even point there will be no additional net profit from combining the system with a desalination plant. We also should note that the average cost of production of the desalinated water in a stand-alone RO plant at the location of the project has been found to be 61.8 ¢/m^3.

An important parameter of every power plant is its guaranteed net power during peak hours. The most critical month is when the possible gap between the demand and the guaranteed power is the highest. In Israel, the demands are usually highest in August and January, with about the same magnitude [12]. Obviously, the critical month in this implementation is January, since it has the lowest guaranteed power compared with the rest of the year. The benefit of combining pumped storage in order to increase the guaranteed power is clearly seen, when, in the combined system, the guaranteed power during peak hours in January is increased by more than 150%.

Prior to this study, the two main reasons that were believed to cause an additional profit from combining the systems were saving in initial investment, and saving in energy. It can be seen in Table 4 that the combination of pumped storage and a desalination plant with an ET does not decrease the initial investment, but increases it, compared to the sum of these systems as stand-alone systems. The main reason for that is the increase of the

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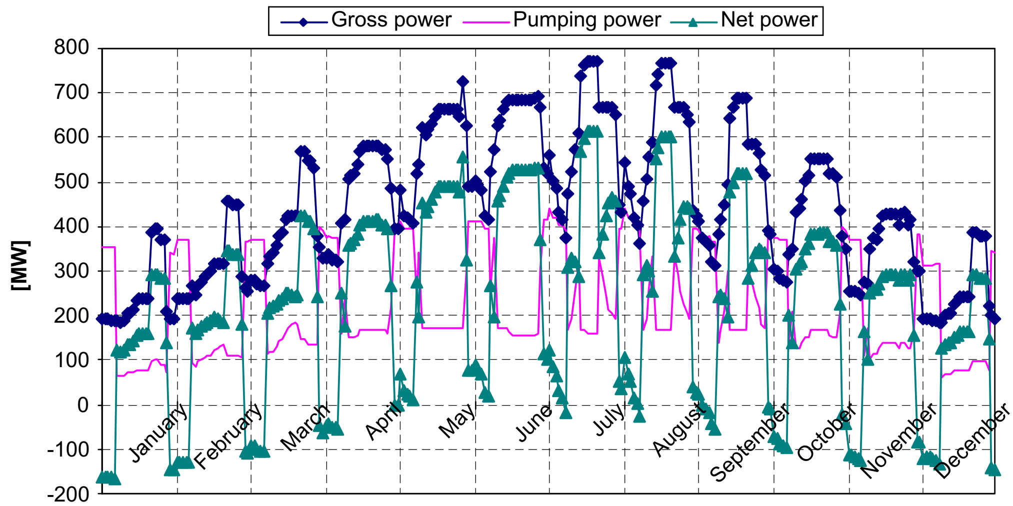

Fig. 6. Combined system: Gross power, Pumping power, and Net power during the short reference year (SRY).

generated power to pumping power ratio of the ET in the combined system. The increase of the attractiveness of the ET leads to a new operating point which includes larger turbines and high head pumps. The increase of these components enables higher net profit, though it requires an additional investment. The energy costs of the combined systems are significantly less than the sum of energy costs of the stand-alone systems, and there is also a saving in the O&M costs, leading to a total decrease of 21.3 m$ (9.2%) in total annual costs, including investment return. The higher net profit in the combined system corresponds to the increase in net energy production. In addition to that, pumping water for the ET is shifted to low-demand hours, and more power is generated during peak hours, which contributes to the additional profit from the combined system. This trend, which is enabled by the use of pumped storage, can be seen in Fig. 6, where the gross power, pumping power and net power are shown for each representative hour during the year. The net power is sometimes negative during low-demand hours (night) due to intensive pumping from the sea to the upper reservoir, which enables high net power during peak demand hours (usually afternoon). The most attractive alternative between these four alternatives is clearly the combined system, for which the annual net profit is 96.8 m$.

6. Summary

1. A problem of the optimal design of a combined system that includes an ET, pumped storage and a desalination plant is formulated and analyzed. The objective function of the optimization algorithm is to maximize annual net profit from the system. The optimization algorithm is based on ''block coordinate descent'', where one step includes a fixation of the diameters of the pipes 1 and 9 (D_1 and D_9), and the other step includes a search of all other variables. The sub problem with the fixed diameters was formulated as a LP problem, where most functions were approximated as linear functions, and a few highly non-linear functions were approximated as piece-wise linear functions. The other subproblem, which includes finding the optimal diameters when all other variables are fixed, is solved analytically by low-dimensional optimization of the objective function.
2. The design of the combined system was found with input parameters that correspond to a potential site near the city of Eilat, Israel. The results of the stand-alone systems were compared with the results of the combined system. The comparison shows that by combining a desalination plant and pumped storage with an ET, there is an additional benefit due to an increase of the energetic efficiency in the combined system, compared to the efficiency of the stand-alone systems. The installation cost of the combined system is not necessarily lower than the sum of installation costs of the stand-alone systems, since sometimes the optimal combination of the systems enables a higher net profit by increasing the sizes of some components in the system. An implementation of the algorithm showed that the additional benefit from combining the systems lead to an increase of 14% in the annual net profit, compared to the sum of profits from the stand-alone systems, when optimally designed. The annual net profit from the combined system was found to be 41% higher than the annual net profit from a stand-alone ET. A new operating point is obtained by combining the systems, which involves an increase of the size of most components in the system, due to an increase of the attractiveness of operating the ET. Another big advantage of the combined system is the increase of the guaranteed power at peak hours in January from 110 [MW] in the stand-alone system to 300 [MW] in the combined system.

Appendix. Mathematical formulation of the LP problems (subproblem 1)

A.1. General form

The general form of the optimization problem is as follows

$$\min_x Z = c^T x$$

subject to :

$$\begin{aligned} &Ax \leqslant b \\ &A_{eq}x = b_{eq} \\ &lb \leqslant x \leqslant ub, \end{aligned} \quad \text{(A.1)}$$

where Z is the objective function to be minimize, c is a vector of costs (c_i is the cost of variable i per unit of the variable), x is a vector of the variables, lb is a vector of the lowest values that can be assigned to the variables and ub is a vector of the highest values that can be assigned to the variables.

The matrix A and the vector b include the parameters of the matrix form of the inequality constraints:

$$\sum_{j=1}^{N_{vars}} a_{ij}x_j \leqslant b_j, \quad i = 1, \ldots, N_{ineq}.$$

The matrix A_{eq} and the vector b_{eq} include the parameters of the matrix form of the equality constraints:

$$\sum_{j=1}^{N_{vars}} a_{eq,ij}x_j = b_{eq,j}, \quad i = 1, \ldots, N_{eq}.$$

Here N_{vars} is the number of variables, N_{ineq} is the number of inequality constraints and N_{eq} is the number of equality constraints.

A.2. Formulation of the LP problems

The following LP problem is actually a compact formulation of four LP problems: LP_1-stand-alone ET, LP_2-stand-alone pumped storage, LP_3-stand-alone desalination plant, LP_4-the combined system. The equations that are relevant to each problem are listed in Table 5. The MOSEK commercial solver [13] was used in order to solve these LP problems.

Table 5
The relevant equations of the LP problems

Problem	Relevant equations
LP_1	(A.3), (A.4), (A.6)–(A.9), (A.12)–(A.17), (A.20)–(A.23), (A.25), (A.26), (A.28), (A.31)
LP_2	(A.3), (A.4)–(A.6), (A.8)–(A.10), (A.14)–(A.18), (A.23), (A.24), (A.26), (A.27), (A.31)
LP_3	(A.3), (A.4), (A.8), (A.14)–(A.16), (A.23), (A.26), (A.31)
LP_4	(A.3)–(A.31)

A.2.1. Variables and limits ($lb_j \leqslant x_j \leqslant ub_j$)

Units: volume [$10^3\,m^3$], power [MW], discharge [m^3/s].

Design variables (see Figs. 2–5):

$0 \leqslant V_i \leqslant_{max,i}$—volume of reservoir i ($i = 1,2,3$) [$1000\,m^3$]

$0 \leqslant P_1 \leqslant \infty$—installed power of pumping station 1 [MW]

$0 \leqslant PREV_{2a} \leqslant \infty$—installed power of the reversible pumps [MW]

$0 \leqslant P_{2a} \leqslant \infty$—installed power of regular pumps 2a [MW]

$0 \leqslant P_{2b} \leqslant \infty$—installed power of pumping station 2b [MW]

$0 \leqslant P_{2c} \leqslant \infty$—installed power of pumping station 2c [MW]

$0 \leqslant Q_i \leqslant \infty$—installed discharge capacity of pipe i ($i = 2,3,4,9,11$)

$0 \leqslant PTUR \leqslant \infty$—installed turbines power [MW]

(Approximated) operation variables (in month m and hour h):

$0 \leqslant v_i(m,h) \leqslant \infty$—water volume in reservoir i ($i = 1,2,3$) [$1000\,m^3$]

$0 \leqslant p_1(m,h) \leqslant \infty$—pumping power of pumping station 1 [MW]

$0 \leqslant p_{2c}(m,h) \leqslant \infty$—pumping power of pumping station 2c [MW]

$0 \leqslant q_{1k}(m,h) \leqslant (Q_1/10)$—the part of the discharge in pipe1 that corresponds to the part k ($k = 1,\ldots,10$) in the piece-wise linear function $p1(q1)$ [m^3/s]

$0 \leqslant q_{2p,m,h} \leqslant \infty$—the discharge from the lower reservoir towards the junction

$0 \leqslant q_{2t,m,h} \leqslant \infty$—the discharge from the junction towards the lower reservoir

$-\infty \leqslant q_3(m,h) \leqslant \infty$—the discharge in pipe 3. Gets a positive sign when the direction of flow is upwards the pipe, and negative when the direction of flow is downwards the pipe

$0 \leqslant q_4(m,h) \leqslant \infty$—the discharge through pumping station 2b

$0 \leqslant q_{11}(m,h) \leqslant \infty$—the discharge in pipe 11

$0 \leqslant q_{9,1}(m,h) \leqslant WD_1$—the part of the spraying discharge that corresponds to the first part in the piece-wise linear function gross_power(q_9)

$0 \leqslant q_{9,i}(m,h) \leqslant WD_i - WD_{i-1}$—the part of the spraying discharge that corresponds to the i-th part in the piece-wise linear function gross_power(q_9), $i = 2,\ldots,14$

$0 \leqslant q_{9,15}(m,h) \leqslant Q_9 - WD_{14}$—the part of the spraying discharge that corresponds to the last part in the piece-wise linear function gross_power(q_9)

A.2.2. Objective function

$$\min Z = \text{annual costs} - \text{annual incomes} = \sum_i Z_i, \quad \text{(A.2)}$$

where Z is the annual profit (with a minus sign), not including constant costs [1000 \$/year] and Z_i are as follows, where $Z_i = c_j x_j$, and the variable x_j is bolded.

Reservoirs installation:

$$Z_1 = \sum_i C_{\mathrm{res}i}\,\mathrm{prRES}_i\,\mathbf{V_i}$$

$$(\mathrm{LP}_1 : i = 1,3; \quad \mathrm{LP}_2 : i = 1,2; \quad \mathrm{LP}_3 : i = 1; \quad \mathrm{LP}_4 : i = 1,2,3). \tag{A.3}$$

Pumping stations installation:

$$Z_2 = \sum_i C_{\mathrm{pump},i}\,\mathrm{pr}_{\mathrm{PUMP}i}\,\mathrm{p}_i$$

$$(\mathrm{LP}_1 : i = 1,2\mathrm{c}; \quad \mathrm{LP}_2 : i = 1; \quad \mathrm{LP}_3 : i = 1,2\mathrm{b}; \quad \mathrm{LP}_4 : i = 1,2\mathrm{a},2\mathrm{b},2\mathrm{c}). \tag{A.4}$$

Reversible pumps installation:

$$Z_3 = C_{\mathrm{rev\_pump}2a}\,\mathrm{prPREV}_{2\mathrm{a}}\,\mathrm{PREV}_{2\mathrm{a}}. \tag{A.5}$$

Pipes installation:

$$Z_4 = \sum_i C_{\mathrm{pipe}}, i_a\, L_i\,\mathrm{prL}_i/10^3\,\mathrm{Q}_i$$

$$(\mathrm{LP}_1 : i = 11; \quad \mathrm{LP}_2 : i = 3; \quad \mathrm{LP}_4 : i = 3,11). \tag{A.6}$$

Turbines installation:

$$Z_5 = C_{\mathrm{tur}}\,\mathrm{prTUR}\,\mathrm{PTUR}. \tag{A.7}$$

Pumping energy for pumping station 1:

$$Z_6 = \sum_m \mathrm{days\_in\_month}(m) \sum_h C_{\mathrm{elec\_cons}}(m,h)\,\mathrm{p}_1(\mathrm{m},\mathrm{h}). \tag{A.8}$$

Pumping energy for supplying the discharge $q2_{\mathrm{p}}$ (either by regular pumps 2a or by the reversible pumps):

$$Z_7 = \sum_m \mathrm{days\_in\_month}(m) \sum_h C_{\mathrm{elec\_cons}}(m,h)\,\mathrm{pump2a}/10^6\,\mathrm{q}_{2p}(\mathrm{m},\mathrm{h}). \tag{A.9}$$

Generated water turbines energy by the reversible pumps (with a minus sign):

$$Z_8 = -\sum_m \mathrm{days\_in\_month}(m) \sum_h C_{\mathrm{elec\_prod}}(m,h)\,\mathrm{tur}2a/10^6\,\mathrm{q}_{2t}(\mathrm{m},\mathrm{h}). \tag{A.10}$$

Pumping energy for pumping station 2b:

$$Z_9 = \sum_m \mathrm{days\_in\_month}(m) \sum_h C_{\mathrm{elec\_cons}}(m,h)\,\mathrm{pump2a}/10^6\,\mathrm{q}_4(\mathrm{m},\mathrm{h}). \tag{A.11}$$

Pumping energy for pumping station 2c:

$$Z_{10} = \sum_m \mathrm{days\_in\_month}(m) \sum_h C_{\mathrm{elec\_cons}}(m,h)/10^3\,P_{2\mathrm{c}}(\mathrm{m},\mathrm{h}). \tag{A.12}$$

Generated power and O&M costs:

$$Z_{11} = \sum_m \mathrm{days\_in\_month}(m) \sum_h \sum_{k=1}^{15} \beta_k[\mathrm{avg\_maint}/10^3 - C_{\mathrm{elec\_prod}}(m,h)]\,\mathrm{q}_{9,k}(\mathrm{m},\mathrm{h}). \tag{A.13}$$

A.2.3. Constraints

Hourly water volumes in the reservoirs cannot exceed the capacities of the reservoirs:

$$v_i(m,h) - V_i \leqslant 0$$

$$(\mathrm{LP}_1 : i = 1,3; \quad \mathrm{LP}_2 : i = 1,2; \quad \mathrm{LP}_3 : i = 1; \quad \mathrm{LP}_4 : i = 1,2,3) \tag{A.14}$$

Hourly pumping power of pumping station i cannot exceed the installed pumping power:

$$p_i(m,h) - P_i \leqslant 0$$

$$(\mathrm{LP}_1 : i = 1,2\mathrm{c}; \quad \mathrm{LP}_2 : i = 1,2\mathrm{a}; \quad \mathrm{LP}_3 : i = 1; \quad \mathrm{LP}_4 : i = 1,2\mathrm{a},2\mathrm{b},2\mathrm{c}). \tag{A.15}$$

Hourly water discharge cannot exceed the discharge capacity:

$$\sum_{k=1}^{10} q_{1,k}(m,h) - Q_1 \leqslant 0, \tag{A.16}$$

$$q_{2p}(m,h) - Q_2 \leqslant 0, \tag{A.17}$$

$$q_{2t}(m,h) - Q_2 \leqslant 0, \tag{A.18}$$

$$q_4(m,h) - Q_4 \leqslant 0, \tag{A.19}$$

$$\sum_{k=1}^{15} q_{9,k}(m,h) - Q_9 \leqslant 0, \tag{A.20}$$

$$q_{11}(m,h) - Q_{11} \leqslant 0. \tag{A.21}$$

The hourly generated power from the ET cannot exceed the installed power of the turbines. The generated power is evaluated from the piece-wise linear function (see [10]):

$$\sum_{k=1}^{15} \beta_k(m,h)\,q_{9,k}(m,h) - \mathrm{PTUR} \leqslant 0. \tag{A.22}$$

Mass balance in the reservoirs:

$$\big|v_1(m,h+1) - v_1(m,h)v + 3.6\,[q_{2p}(m,h) - q_{2t}(m,h) - \sum_{k=1}^{10} q_{1,k}(m,h)] = -3.6 \times 0.5\,\mathrm{evap\_disch}, \quad m = 1,\ldots,12 \quad h = 1,\ldots,24, \tag{A.23}$$

$$v_2(m,h+1) - v_2(m,h) + 3.6\,q_3(m,h) = -3.6 \times 0.5\,\mathrm{evap\_disch}, \quad m = 1,\ldots,12, \quad h = 1,\ldots,24,$$

$$v_3(m,h+1) - v_3(m,h) + 3.6\,[q_{11}(m,h) - \sum_{k=1}^{15} \text{brine\_coef}_k(m,h)\, q_{9,k}(m,h)] = 0,$$
$$m = 1,\ldots,12 \quad h = 1,\ldots,24. \tag{A.25}$$

Mass balance in the reservoirs between midnight and 1:00 during a month:

$$v_1(m,1) - v_1(m,24) + 3.6\,[q_{2p}(m,24) - q_{2t}(m,24) - \sum_{k=1}^{10} q_{1,k}(m,24)] = -3.6 \times 0.5\,\text{evap\_disch}, \quad m = 1,\ldots,12, \tag{A.26}$$

$$v_2(m,1) - v_2(m,24) + 3.6\,q_3(m,24) = -3.6 \times 0.5\,\text{evap\_disch}, \quad m = 1,\ldots,12, \tag{A.27}$$

$$v_3(m,1) - v_3(m,24) + 3.6\,[q_{11}(m,24) - \sum_{k=1}^{15} \text{brine\_coef}_k(m,24)\, q_{9,k}(m,24)] = 0$$
$$m = 1,\ldots,12, \tag{A.28}$$

Mass balance (Kirchoff 1) in the pumped storage junction (see Fig. 5):

$$q_{2p}(m,h) - q_{2t}(m,h) - q_3(m,h) - q_4(m,h) = 0,$$
$$m = 1,\ldots,12, \quad h = 1,\ldots,24. \tag{A.29}$$

Mass balance (Kirchoff 1) in the desalination plant junction (see Fig. 5):

$$\sum_{k=1}^{15} q_{9,k}(m,h) - q_4(m,h) = Q_{\text{des\_sec}}(m,h),$$
$$m = 1,\ldots,12, \quad h = 1,\ldots,24. \tag{A.30}$$

The piece-wise linear function of the pumping power of pumping station i (see [10]):

$$p_i(m,h) - \sum_{k=1}^{N_i} \gamma_{i,k}\, q_{i,k}(m,h) = 0$$
$$(\text{LP}_1 : i = 1, 2c; \quad \text{LP}_2 : i = 1, 2a;$$
$$\text{LP}_3 : i = 1; \quad \text{LP}_4 : i = 1, 2a, 2b, 2c). \tag{A.31}$$

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