Calculations
for an
Implementation of the Absorption Cooled Energy Tower

Robert J. Rohatensky

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Southern Saskatchewan, Canada

Moderate Climate, Average Humidity, 6 months of sub-zero C temperatures

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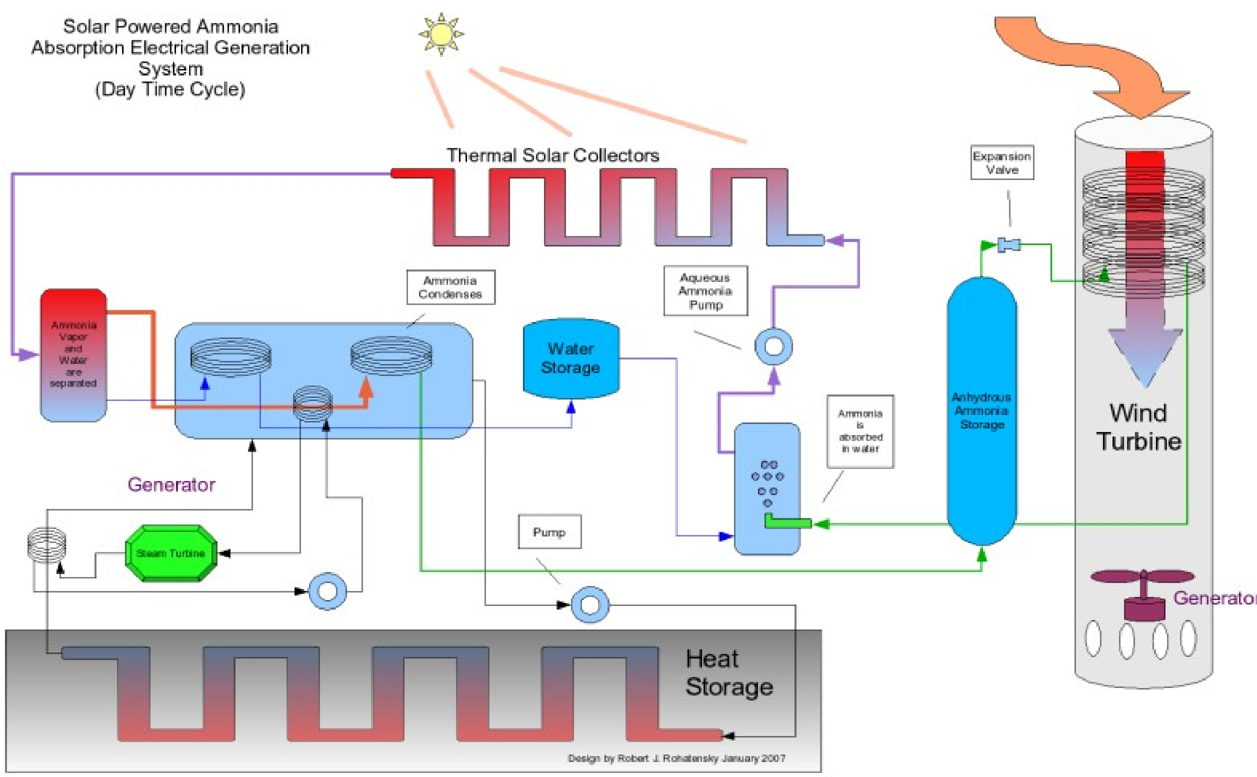
Overview

The climate, annual solar radiation and economic need for this location would best be suited by an bi-directional tower with very large seasonal heat storage. A major portion of the energy used during winter months is structure heating and in an agricultural economy shifted towards electricity, the major electrical power requirements are during the summer months.

The tower structure would be hourglass shaped to increase air velocity through the wind turbine and include very large seasonal thermal storage integrated possibly integrated into methane bio-gas production, with the gas burned in the system. The intake and outlet would rotate to prevailing winds. The latitude and low land cost lends itself to large trough solar collectors operated only through the 6 warmer months. The area also has large supplies of coal and natural gas which could be integrated into the system to increase reliability and the supplementary use of fossil fuels with the solar system would allow for high efficiency of the fossil fuel usage.

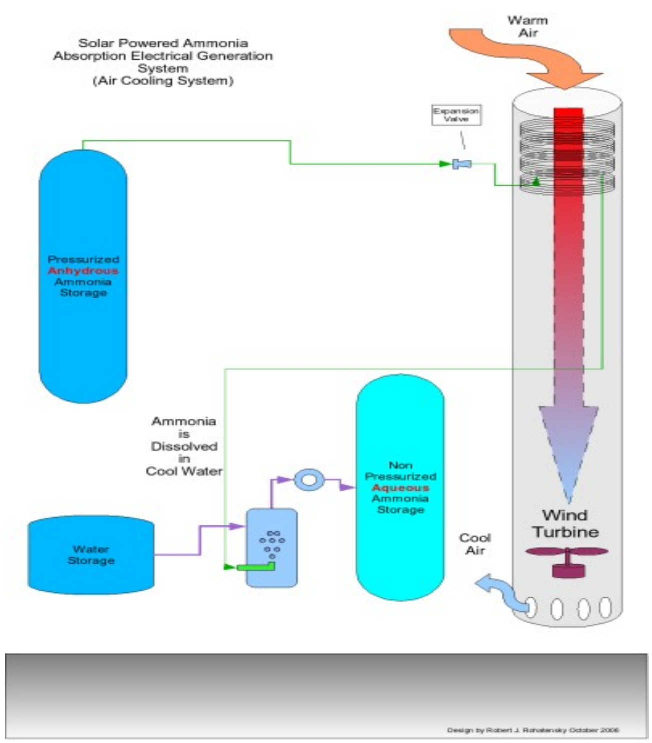
Summer daytime system

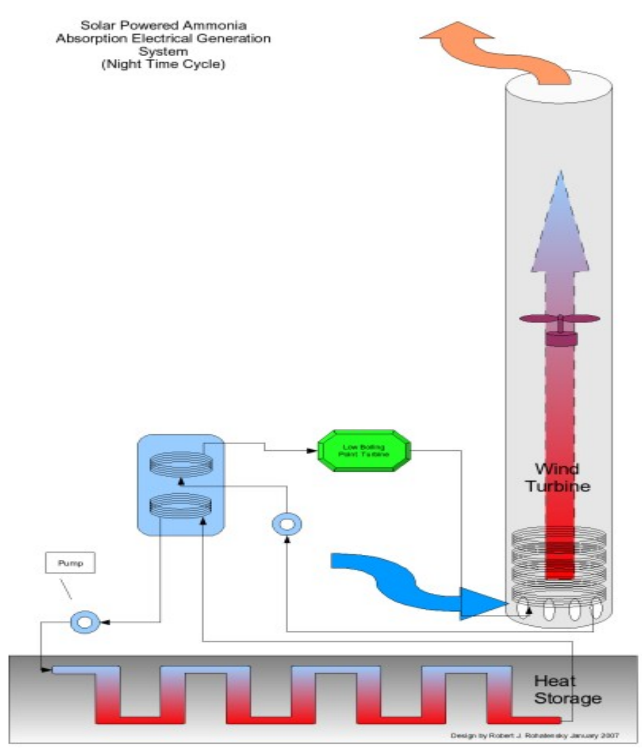


Summer Night and Winter System

There is a question at this location how the system should function at night during the summer months. The seasonal thermal storage has a high value, but the air in the warmest summer months doesn't cool off drastically. The system can either continue to cool the air on summer nights storing the aqueous ammonia until day or it can reverse and operate in "winter" mode at night. This would probably be dependent on the actual temperature on a given night in summer and in spring and fall there is substantial night time temperature drop. If the system operates in downdraft mode at night is requires much more ammonia storage.

Downdraft Mode Summer Night Option A	**Updraft Mode** Winter Operation and Summer Night Option B





Winter Operation

The winter operation is very similar to existing low gradient geothermal utilizing a low boiling point fluid steam turbine. There are 3 major efficiency improvements over existing systems like at Chena Hot Springs.

1. The heat source is very close and has low pumping cost over deep geothermal.
2. The turbine is air cooled and the heat from the cooling causes an updraft in the tower.
3. Some additional energy is recovered in the wind turbine. This location has 6 months of sub-zero temperatures and many days of -25°C which allows for a large temperature gradient between the stored heat and the ambient air.

NASA Surface meteorology and Solar Energy
At Latitude 50 and Longitude -104
Average elevation: 584 meters

Monthly Averaged Insolation Incident On A Horizontal Surface (kWh/m2/day)

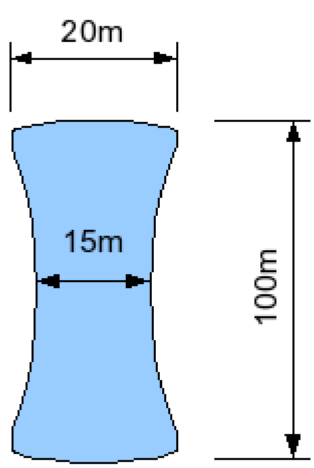
Lat 50 Lon -104	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual Average
10-year Average	1.17	2.06	3.1	4.63	5.4	5.78	6.08	5.05	3.43	2.29	1.28	0.96	3.44

Monthly Averaged Air Temperature At 10 m Above The Surface Of The Earth (° C)

Lat 50 Lon -104	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual Average
10-year Average	-17.4	-14.3	-6.92	2.27	11.1	16.6	20	19.2	10.7	1.51	-12.1	-19.9	0.99

Due to the low temperature and insolation levels during the winter, the solar collection system would only be operated from April through September. The 6 month average for these months is 5.06 kWh/m2 insolation incident and 13.31 C average temperature.

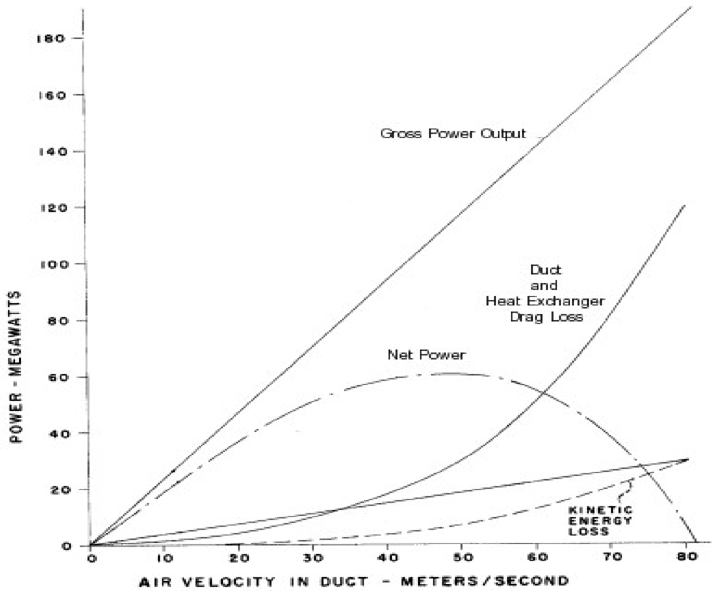
The power output of the wind turbine in the tower is based on the air velocity and the diameter of the tower. The air velocity is based on the buoyancy of the air external to the tower, the drag loss across the heat exchanger and tower walls and the exit loss. The buoyancy difference is based on the air temperature and relative humidity difference. Because there is no energy expended in expanding the pressurized ammonia in the cooling coils, the power output of the system is based on the height and diameter of the tower, the surface area of the heat exchanger and the ambient air temperature and humidity.



Assuming a medium scale system with a tower height of 100m and a diameter of 20m narrowing to 15m in the center. The tower will function bi-directionally seasonally and the wind turbine would be located at the center of the tower with a diameter of 15m.

Wind Turbine Power Calculation (Summer Months)

This chart from patent US003894393 for the water spray tower has a peak net output with a wind velocity of 50 m/s. The ambient air temperature and tower height are too low to achieve this air velocity, but a sustained 20 m/s (72 km/h) is a reasonable target air velocity. The system will be designed to control the ammonia flow and the resulting air cooling to keep the air velocity in the tower at a constant rate. A constant air flow will allow the system to have a reliable turbine rotation speed and to utilize AC alternators at a fixed frequency rather than DC systems and inverters for AC output. This lowers the cost of the generating system substantially over traditional wind turbines with varying RPM and DC generators.



Target Air Velocity: 20m/s

Area of turbine: $A=\pi r^2$
$A=7.5^2 \times 3.14159$
$176.71 m^2 - \text{hub area} = 150 m^2$

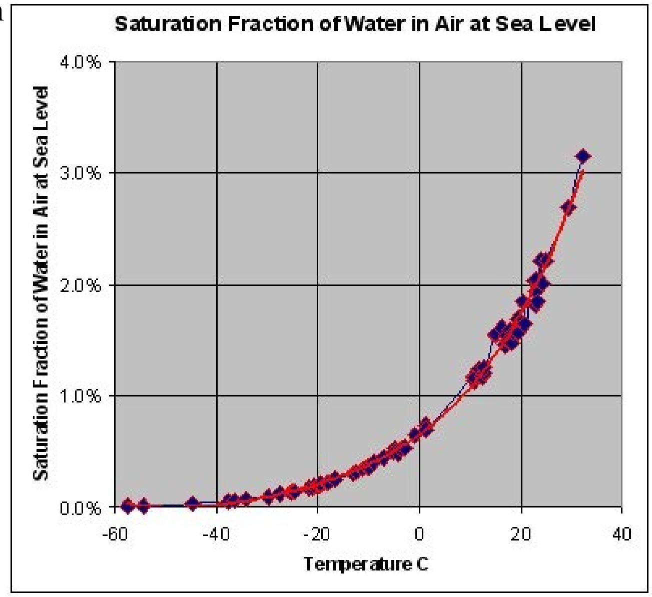
Air volume through turbine: $V_a=U \times A$
$20 m/s \times 150 m^2 = 3000 m^3/s$

To achieve the desired 20 m/s air velocity 3000 cubic meters of air per second need to be moving through the turbine. The top and bottom of the tower are wider so although the same volume of air is moving into the top of the tower the air velocity is lower through the larger area. The exit loss where the output air meets the external air is reduced by having a larger area and lower velocity.

Area of top inlet: $A=\pi r^2$
$A=3.14159 \times 10^2$
$314.59 m^2 - \text{heat exchanger area} = 250 m^2$
$V_a=U \times A$
$3000 m^3/s \div 250 m^2 = 12 m/s$

This assumes a heat exchanger horizontal surface area of ~ 64 m². Constructing the tower in an hourglass shape allows for the surface area of the inlet not to impede the airflow and cause a vacuum restriction through the heat exchangers.

To goal is to remove the required amount of heat from the air at the top of the tower to cause the negative buoyancy to achieve the required air velocity across the tower height. In this model the condensed air is allowed to drip off the cooling coils and is ignored in the calculation.



The formula for air density is:

$$\rho = \frac{p}{R \times T}$$

The specific gas constant "R" for dry air is:

$$R_{dry,air} = 287.05 \frac{J}{kg \times K}$$

The target is to lower the 15°C average ambient air to 0°C.
For dry air the density at 93.350 kPa at 15°C:

$$\frac{93350/287.05}{288.15} = 1.128\, kg/m^3$$

For 15°C air at 40% relative humidity, the air is approximately 0.6% water vapor with a density of 0.804 kg/m^3. This offsets the density by 0.804*.006:

$$1.191 \times 0.994 + 0.804 * 0.006 = 1.189\, kg/m^3$$

For dry air the density at 93.350 k Pa at 0°C:

$$\frac{93350/287.05}{273.15} = 1.191\, kg/m^3$$

Although the relative humidity of the air in the tower after cooling is 100% (below outside air dew point) the 0°C temperature means that the absolute humidity is near the same as the 40% relative humidity input air at around 0.06% and the water vapor density offset is near the same as the warm input air.

$$1.128 \times 0.994 + 0.804 \times 0.006 = 1.121\, kg/m^3$$

	External	Internal
External Air Temperature	15°C (288.15°K)	0°C (273.15°K)
Relative Humidity	40%	100%
Air Pressure (586m + 100m above sea level)	93.350 k Pa	93.350 k Pa
Air Density ρ	1.121 kg/m^3	1.189 kg/m^3

The volume of the air inside the tower is 2 times the volume of a truncated cone:

$$V = \Pi (R^2 + rR + r^2) h/3 \times 2$$
$$V = \Pi \times (10^2 + 10 \times 7.5 + 7.5^2) \times 50 \times 2$$
$$V = 72649.33 m^3$$

The available potential energy is the negative buoyancy of the denser cold air inside the tower relative to the air outside the tower.

$$F = V \times (\rho_i - \rho_o) \times 9.8$$
$$F = 72649.33 \times 1.189 - 72649.33 \times 1.121 \times 9.8$$
$$F = 4940.15 \times 9.8$$
$$F = 48413.5 N$$

The average velocity of the air moving down the tower without drag:

$$v_a = \frac{\sqrt{2gd}}{2}$$
$$V_a = \frac{\sqrt{2 \times 9.8 m/s^2 \times 100 m}}{2}$$
$$V_a = 22.14 m/s$$

The peak velocity of the air after it falls 100m (excluding exit loss):

$$v_a = \sqrt{2gd}$$
$$V_a = \sqrt{2 \times 9.8 m/s^2 \times 100 m}$$
$$V_a = 44.27 m/s$$

The assumption is that although the wind turbine is located in the center of the tower vertically, and the air velocity at the center would be ½ of the velocity at the bottom of the tower (if there was no exit loss). The exit loss is substantial and is caused by the downdraft air having to push the static air at the bottom. With exit loss, the real velocity of the air will be much less than the peak velocity. This is complicated to understand where the ideal location for the turbine would be, but the bi-directional nature of the design lends to having the turbine in the vertical center of the tower so the average velocity of the air is used in the calculations and it is assumed that this is a reasonable estimate.

The gross power is the flow rate in kg/s (of buoyancy) times 9.8 times height.

$$W = \text{flow rate} \cdot g \cdot h$$
$$W = (1.189 - 1.121) \times 3000 \times 9.8 \times 100$$
$$W = 199920 = 200\, kW \text{ gross power}$$

Or using a wind turbine formula based on the 22.14 m/s wind speed with a 0.33 turbine power efficiency the actual power is:

Power delivered = Cp x area of wind turbine x $^1/_2\ \rho v^3$

$$P = 0.33 \times 150 \times \frac{1}{2} \times 1.189 \times 22.14^3$$
$$P = 320\, kW$$

In other words, the power available is the difference in density (total mass) in the cold air inside the tower relative to the air outside the tower and how fast it is falling. The efficiency of the wind turbine is much better than a conventional natural wind turbine due to much lower tip loss because the turbine in a duct. The assumption is that with a well designed turbine much more of the energy could be converted to electricity. The air in the tower can also be routed into a vortex to increase the angle of attack and efficiency.

The electrical power output of the wind turbine in the tower is ~ 200kW consistently 24 hours per day.

Alternate Flow Calculations Using the Stack Effect Formula

Another method of calculating the air flow through the tower is by using the Stack Effect formula:

$$Q = C A \sqrt{2 g h \frac{T_h - T_c}{T_h}}$$

Q = stack effect flow rate, m^3/s
A = flow area, m^2
C = discharge coefficient (usually taken to be from 0.65 to 0.70)
g = gravitational acceleration, 9.8 m/s^2
h = height, m
T_h = warm temperature, K
T_c = cold temperature, K

This will give the flow rate in m^3/s, but doesn't take into account differences in absolute humidity.

$$0.65 * 314 * \sqrt{2 * 9.8 * 100 * \frac{(288.15 - 273.15)}{288.15}} = 2061 m^3/s$$

The exact calculation of the power output of the wind turbine is complicated and there are a lot of fluid dynamics involved. The Second Law of Thermodynamics limits the amount of power out of the wind turbine to the amount of power put into the heat pump minus efficiency loss. An estimate of < 50% of the solar energy input of the heat pump should be reasonable for the electrical power output of the wind turbine.

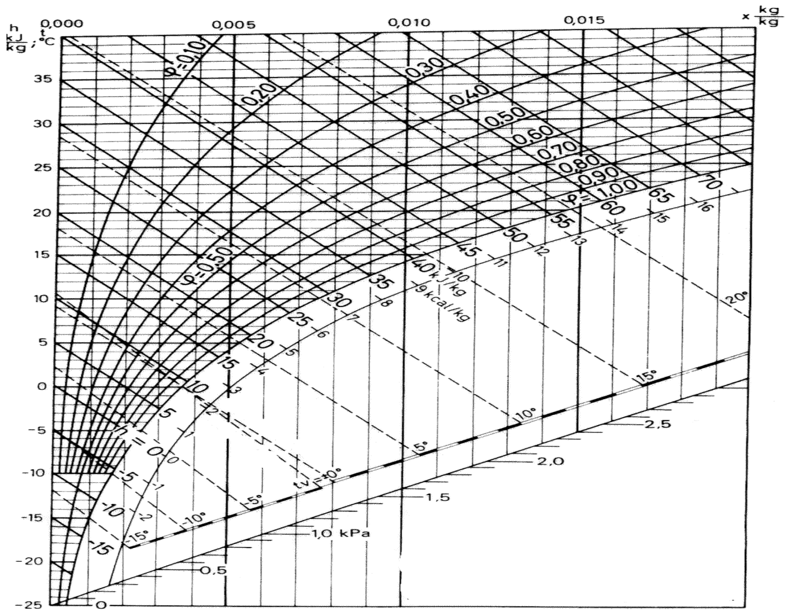
Estimated Electrical power output of wind turbine

24h X 200kW = 4800 kWh/day

(average during the 6 months of “summer")

Air Cooling Calculation (Summer Months)

Mollier Diagram



Based with an ambient air temperature of 15°C and relative humidity of 40% across the tower height of 100m to lower the temperature of $3000m^3/s$ of air requires removal of 15° X $3000m^3$ X 1.186 kg/m^3 with air having a specific heat of 1.012 J per g per K or 1012 J per Kg per K

1 m^3/s of air at 15^oC and relative humidity 40% (A) is cooled down to 0^oC (B). The surface temperature of the cooling coil is -20^oC (C). The density of air at 15^oC is 1.186 kg/m^3.

Using the Mollier diagram the state of the cooled air (B) is in the intersection between the straight line between (A) and (C) and the 15^oC temperature line.

From the Mollier diagram the enthalpy in (A) is 25 kJ/kg, in (B) 8.5 kJ/kg and in (C) -15 kJ/kg.
The Contact Factor can be calculated as:

$$\beta=((25\,kJ/kg)-(8.5\,kJ/kg))/((25\,kJ/kg)-(-15\,kJ/kg))$$
$$=0.4125$$

The total heat flow can be calculated as:

$$q = (1\,m3/s)(1.186\,kg/m3)((25\,kJ/kg) - (8.5\,kJ/kg))$$
$$= 19.569\,(kJ/s, kW)$$

The sensible heat flow can be calculated as:

$$q_s = (1\,m3/s)(1.186\,kg/m3)(1.01\,kJ/kg.oC)(15\text{oC} - 0\text{oC})$$
$$= 18.3\,(kW)$$

According to the Mollier diagram the specific humidity in (A) is 0.096 kg/kg and in (B) 0.001 kg/kg and the latent heat flow can be calculated as:

$$q_s = (1\,m3/s)(1.186\,kg/m3)(2{,}502\,kJ/kg)((0.004\,kg/kg) - (0.003\,kg/kg))$$
$$= 2.9\,(kW)$$

At 3000m3/s the total heat removed from the air is:

$$19 \times 3000\text{m}^3/s$$
$$= 57{,}000{,}000\,j/s$$
$$= 57{,}010\,kW$$
$$= 57\,MW$$

To drop the 3000m^3/s of air by 15°C requires approximately 19 kW/m^3

=

57 MW of heat removed from the ambient air moving at 3000 m^3/s

Pressurized Anhydrous Ammonia Requirements

The heat capacity of ammonia vapor is 35.06 J/mol K with molar mass of 17.0304 g/mol.

$$\frac{35.06}{17.0304} = 2.059\, J/g\, K$$

The latent heat of ammonia is 1369 J/g K with a boiling point at atmospheric pressure of -33°C.

To remove 57 MW of heat with liquid ammonia changing state and being warmed to -13°C, the latent heat is approximately 600x the specific heat.
To raise ammonia vapor 20°K it takes 2.059 J X 20 /g = 41.2 Joules per gram.

$$g/s \times specific\ heat \times K + g/s * latent\ heat$$
$$54{,}147{,}060 = Q \times 20 \times 2.059 + Q \times 1369$$
$$\frac{54{,}147{,}060}{Q} = 41.2 + 1369$$
$$\frac{54{,}147{,}060}{Q} = 1410.2$$
$$Q = 38{,}396\, g/s$$

To remove 57 MW of heat from the air:
38 kg/s of liquid ammonia must be evaporated and heated 20°K (to -13°C)

Liquid ammonia density is 681.91 g/L
The pressurized liquid ammonia input is 38,396g/s at 681.91 g/L

$$38{,}396/681.91 = 56.3\, L/s$$

56.3 litres/s of liquid ammonia to transfer 57MW of heat from 3000m²/s of air

Operating 24 hours per day is a total consumption of 56 x 24 x 60 x 60 = 5 million litres per day of liquid ammonia. This is only in the case of "cooling" the air at night. If the system is reversed at night the 56.3 L/s of ammonia would only be used during the day.

Ammonia Recovery System

The solar collectors would need to boil enough aqueous ammonia to meet the 57.83 L/s constant intake during sunlight hours.

Ammonia dissolves in water at 89.9 g/100 ml at 0C

To absorb the ammonia vapor at the 38 kg/s rate is 38396 / 899 = 42.7 l / s of cold water. The absorption of ammonia in water is exothermic (gives off heat), as the ammonia vapor is dissolved the aqueous ammonia solution increases in temperature and absorbs all of the heat energy in the ammonia vapor. The temperature of the water increases and the aqueous ammonia remains at it's vapor point. If there is no increase in pressure, any increase in aqueous ammonia temperature will cause the ammonia to boil.

The aqueous ammonia is pumped to approximately 200 psi pressure prior to entering the heating stage. The pump uses energy but substantially less than a compressor due to the smaller volume. The amount of heat to extract the ammonia is the latent heat of the output ammonia which is 5 million litres or 3.4 million kg per day plus the heat required to raise the water the same temperature. The aqueous ammonia solution is at approximately 50% and should be at the vapor point at all times.

The heat to boil off the ammonia is:

$$\begin{aligned} &= 38{,}396\, g/s \times 1369\, J/g\,K \times 24\text{h} \\ &= 52.5\, MJ/s\,,\, MW\,(24\text{h}) \\ &= 1{,}261\, MWh\ per\ day \end{aligned}$$

Local Solar radiation average 5kWh/m2.

$$1{,}261{,}538\, kWh/5 = 252{,}307\, m^2\ of\ thermal\ solar\ panel$$

Solar panel daily input

1.3GWh$_{(t)}$ / day solar thermal input
(Assuming the air is “cooled” with Night Option A)

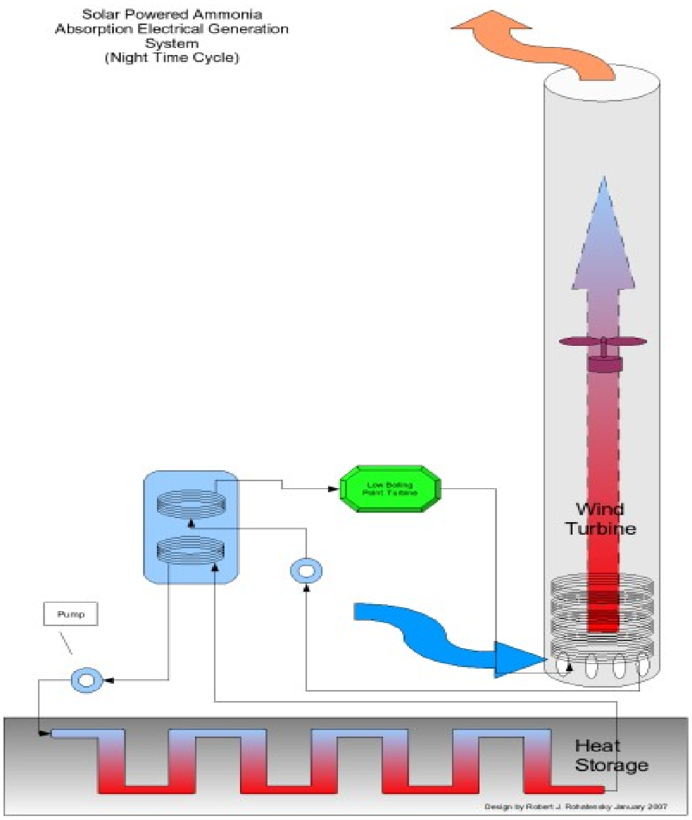
Winter (and Night Option B) System Power Calculation

The winter system is very similar to a conventional geothermal with the addition of air cooling. In a Western Canadian climate arid location the air is below 0°C for six months of the year with many days of < -20°C air temperature.

The temperature used for the winter power calculation is -10°C the average daily temperature of the 6 coldest months.

In this system during the summer 57MW of heat is removed from the air and 108MW of heat is collected in the solar collectors daily for the 6 warmer months. All of this heat is stored in the underground thermal storage.

$$H = \text{daily thermal storage}$$
$$H = 57MW * 24 + 1.3GWh$$
$$H = 2.66\, GWh_{(t)} / day * 180\, days$$



Depending on the design of the thermal storage there will be losses with heat dissipating into the earth, as an estimate 50% of the stored heat can be used at night. The intent of the night system is to generate some power while "cooling off" the thermal storage.

The average temperature of the thermal storage is difficult to calculate without knowing the material and transfer system. The input heat from the solar collectors could approach 100°C, but the input heat from condensing the ammonia will be lower.

The system functions like existing low-gradient geothermal, with low transfer media energy cost due to the very close location.

Although it would be possible to calculate the geothermal turbine, tower airflow and power output separately, the system can be treated as a whole and it is just a steam engine.

If the thermal storage is 70°C and the ambient air is -10°C, the maximum Carnot efficiency of the whole system (both the geothermal and wind turbine) is:

$$\text{Maximum Carnot Efficiency for Complete Night System}$$
$$\eta = 1 - \frac{T_C}{T_H}$$
$$\eta = 1 - \frac{(-10 + 273.15)}{(70 + 273.15)}$$
$$\eta = 0.23$$

There is 2.66GWh$_t$ per day of thermal storage stored over the 6 warmer months. With an assumption that the winter system can convert 10% of the maximum Carnot (2% of thermal) into electricity and accounting for a 50% loss of heat from the thermal storage.

$$P = 2.66\text{GW}_{(t)} \cdot 50\% \cdot 2\%$$
$$P = 1330\text{M}_{(t)} \cdot 2\%$$
$$P = 26\,MWh_{(e)}\,per\ day$$

Winter Complete System Electrical Power Output

26 MWh$_{(e)}$ per day

The added benefit is that by spring the winter system with many day of -20°C will have cooled off the thermal storage much more than the +5°C natural shallow earth temperature.

System Losses

There are losses in the system in pumping the aqueous ammonia, cycling the working fluid in the thermal storage and thermal collection, but the electrical power output is constant and much higher than any direct solar system with an output of > 100% of the solar input.

The thermal storage is the major gain in moderate climates like Canada and the system would produce 2.66 GWh/day of thermal storage from 1.3GWh/day of solar collectors. This would only be sustainable while the thermal storage is colder than ambient air and then it would drop off to a 1:1 ratio with the solar collector thermal input, but this is only limited by the size of the underground thermal storage.

Once the system is in place, the winter cycle will cool off the thermal storage much more than it is naturally. With ambient winter temperatures dropping to -30°C, the underground thermal storage can be cooled off substantially below the natural +5°C that occurs below the frost line. With a well planned seasonal thermal storage system it would be possible to have a very large mass with sub-zero C temperatures stored for spring.

Summer Electrical and Thermal Output

Wind turbine output = 200 kW x 24h =

4.8 MWh(e) per day

Total gross thermal storage = 57 $MW_{(t)}$ * 24 h + 132 $MWh_{(t)}$

2.66 $Gwh_{(t)}$ per day of thermal storage

The second heat recovery turbine during the summer should convert 2% of the the heat going into thermal storage into electricity.

2.66$GWh_{(t)}$ x 2% = 53 MWh(e) per day summer

26 $MWh_{(e)}$ per day winter

(during daylight hours)

The winter cycle power output would depend on the volume of heat extracted from the thermal storage per day. There is massive thermal storage with 2.6 $GWh_{(t)}$ / day going into thermal storage during the summer and substantial cold ambient air during the winter.

Total System Statistics:

- Summer wind turbine electrical output: **4.8 MWh / day** (constant)
- Summer heat recovery turbine output: **53MWh / day**
- Total summer electrical output: **58 MWh / day**
- Winter electrical output: **26 MWh / day**
- Thermal storage: **2.6GWh of heat / day** (summer)
- 100 m tower
- 250,000 m^2 trough concentrated solar collectors
- large thermal storage (vertical bore-hole 100m deep 1km x 1km= 100,000,000 m^3)
- 5,000,000 litres pressurized anhydrous ammonia storage
- 5,000,000 litres water storage
- 5,000,000 litres aqueous ammonia storage